CS 577: Introduction to Algorithms	Homework 7
Out: 11/10/10	**Due: 11/17/10**

Ground Rules

- **Grading.** You will be graded on the correctness as well as clarity of your solutions. You are required to prove any claims that you make. In particular, when you are asked to design an algorithm, you must argue its correctness as well as running time. You may use without proof any theorems proven in class or in the textbook, as long as you state and cite them properly.
- **Collaboration.** You may work on and submit solutions in pairs.
- **Lateness.** Homework is due promptly at the start of class. Late homework will receive zero credit.
- **Extra credit questions.** Extra credit questions will not directly contribute to your score. However, they will be taken into account in the final grading and may improve your overall grade if your total score is close to the boundary between two grades. Furthermore they are fun to solve and improve your understanding of the course.
- Start working on your homework early. Plan your work in such a way that you have the opportunity to put some problems on the back burner for a while and revisit them later. Good luck!

Problems

1. **(4 points.)** For this question, you are given a directed network $G = (V, E)$ with capacities c_e on edges and a source-sink pair (s, t). You are also given the max s-t flow in the graph, f^* specifying the amount of flow on every edge in the graph. An edge in the network is called a *critical* edge if reducing its capacity by one unit decreases the s-t max flow in the network.

   Develop an algorithm for determining all the critical edges in the given graph. Your algorithm is allowed to use the max flow f^* and should run in time $O(mn)$.

   (Hint: try your algorithm on the example in HW6.)

2. **(4 points.)** Problem 7.12 in the textbook (Pg. 420).

3. **(7 points.)** Problem 7.27 in the textbook (Pg. 431).

   Hint: As you may have guessed, you should reduce this problem to a network flow problem. Prove that a fair schedule exists if and only if there exists an integral max flow in your network of a certain size. Proceed to show (for part (a)) that the max flow has the right size. Note that in order to prove this, you only need to display a fractional flow of that size, and not necessarily an integral one (Ford-Fulkerson guarantees the existence of an integral flow of the same size).

4. **(Extra credit.)** Recall that a *vertex cover* of a graph $G = (V, E)$ is a subset of V, V', such that *every* edge in E is incident on some vertex in V', that is, for each edge $(u, v) \in E$, one or both of u and v are in S. Show that the problem of finding the minimum vertex cover in a *bipartite* graph reduces to maximum flow. (Hint: Can you relate this problem to the minimum cut in an appropriate network?)