CS 577: Introduction to Algorithms	10/30/2012
Homework 4	
Instructors: Shuchi Chawla and Dieter van Melkebeek	Due: 11/13/2012

This assignment covers network flow (chapter 7 in the textbook). See the first assignment for guidelines.

Ungraded problems

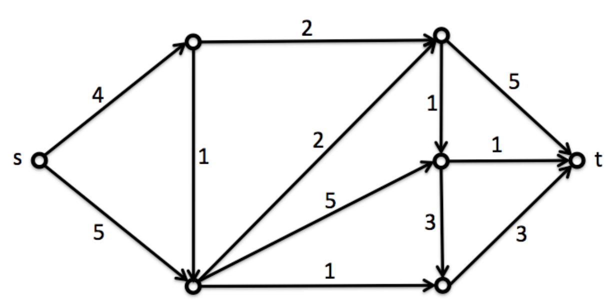
1. Problem 7.27 in the textbook (p. 431).
2. A *vertex cover* of a graph $G = (V, E)$ is a subset S of V such that *every* edge in E is incident on some vertex in S, that is, for each edge $(u, v) \in E$, one or both of u and v are in S.

   Show that for bipartite graphs G, the minimum size of a vertex cover equals the maximum size of a matching.
3. Problem 7.42 in the textbook (p. 443).

Graded problems

4. (5 points) An edge in a network is called *upper-binding* if increasing its capacity by one unit increases the maximum flow in the network. An edge is called *lower-binding* if reducing its capacity by one unit decreases the maximum flow in the network.

   (a) For the network G below determine a maximum flow f^*, the residual network G_{f^*}, and a minimum cut. Also identify all of the upper-binding edges and all of the lower-binding edges.

   (b) Develop an algorithm for finding all the upper-binding edges in a network G when given G and a maximum flow f^* in G. Your algorithm should run in linear time.

   (c) Develop an algorithm for finding all the lower-binding edges in a network G when given G and a maximum flow f^* in G. Your algorithm should run in time $O(mn)$.
5. (5 points) Problem 7.31 in the textbook (p. 434-435).
6. (5 points) Problem 7.46 in the textbook (p. 444).