CS 577: Introduction to Algorithms (Section 1)	Homework 6
Out: 11/26/14	**Due: 12/12/14**

Ground Rules

See homework 1.

Ungraded problems

1. Problem 8.01 in the textbook (Pg. 505).
2. Problem 8.24 in the textbook (Pg. 517-18).
3. Problem 8.27 in the textbook (Pg. 518-19).

Graded problems

4. **(5 points)** Consider the following variant of 3-SAT: Given a 3-CNF formula, decide whether there exists an assignment to the variables of the formula such that every clause contains at least one literal that is true and at least one literal that is false. Show that this problem, *Not-All-Equal-SAT (NAE-SAT)*, is NP-complete.

   (Hint: Use Circuit-SAT in your reduction.)

5. **(5 points)** In the *Max-Cut* problem, our goal is to find a partition of the vertex set of an unweighted graph with the maximum number of edges going across. Thus, Max-Cut is the opposite of (global) Min-Cut. Surprisingly (as you will show in this question), the complexity of finding a Max-Cut is very different from that of finding a Min-Cut!

   Formally, the decision version of the problem is stated as follows:

   Given a graph G with edge set E and vertex set V, and a number k, does there exist a partition of the vertices into two sets V_1 and V_2 with $V_1, V_2 \neq \emptyset$ and $V_1 \cup V_2 = V$, such that $|(V_1 \times V_2) \cap E| \geq k$.

   Prove that the Max-Cut problem is NP-complete.

   (Hint: Use NAE-SAT from the previous problem in your reduction. You may assume that NAE-SAT is NP-complete.)

6. **(5 points)** Problem 8.41 in the textbook (Pg. 528).

7. **(Extra Credit)** In this question we consider the special case of the Max-Cut problem with $k = |E|/2$. Surprisingly a $1/2$-Cut, that is, a cut with at least $|E|/2$ edges, always exists. Give a polynomial time algorithm for finding such a $1/2$-Cut.