CS 787: Advanced Algorithms	**Final Exam**
Out: 12/06/15	**Due: 12/13/15**

Guidelines

- This exam consists of 3 problems, all worth equal points. You are required to solve all of the problems.
- The exam should be done and submitted individually.
- You are allowed to consult any sources discussed in class or posted on the class webpage or Piazza. Please refrain from using any other sources.
- Please prove all claims you make, or cite an appropriate reference.
- Please typeset your solutions, and write your name clearly on your submission.

Problems

1. Given a point (x, y) in the unit square $[0, 1]^2$, we define the "shadow" of the point, $S(x, y)$, to be the set $[0, x] \times [0, y]$ of points that are to the bottom left of (x, y) in the unit square. The shadow of a collection C of points is the union of the shadows of points in the collection: $S(C) = \cup_{(x,y)\in C} S(x, y)$. Given a collection C, we call a point $(x, y) \in C$ "unshadowed" if $(x, y) \notin S(C \setminus \{(x, y)\})$. (See the example below.)

   (a) Consider picking n points uniformly at random from the unit square, and call this set of points C. (We can do this by picking the x-coordinate of every point independently uniformly at random from $[0, 1]$ and y-coordinate similarly.) Determine the expected number of unshadowed points in the collection C.

   (b) Given a collection C with n points, and a number $k < n$, develop a polynomial time algorithm for picking a subset $C' \subset C$ with $|C'| \leq k$ such that the area of $S(C')$ is maximized.

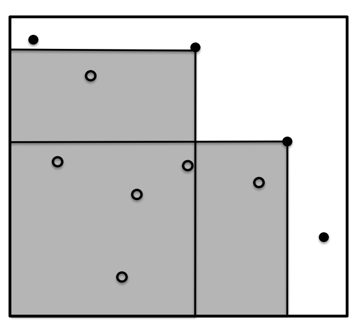

Figure 1: This example shows a collection of 10 points in the unit square. Of these the four dark points are unshadowed. The shaded area represents the shadow of the middle two unshadowed points. The two points form a solution to question 1(b) for this collection with $n = 10$ and $k = 2$.

2. Let $G = (V, E)$ be a d-regular graph. A local (L, Δ)-code on G is a function f mapping the vertices of G to $\{0, 1\}^L$ such that for every $(u, v) \in E$, the Hamming distance between $f(u)$ and $f(v)$, that is the number of coordinates that the two codes differ in, is at least Δ. Show that there exist constants $a, b \geq 1$ such that for every d-regular graph G, there exists a local (L, Δ)-code on G with $L = a \log d$ and $\Delta = \frac{\log d}{b}$.

3. In this problem you will develop an approximation algorithm for MAX-SAT. Recall that in MAX-SAT you are given a CNF formula ϕ. Let $\mathcal{C}$ denote the set of clauses in this formula and V denote the set of variables in the formula. The goal is to find an assignment for the variables that satisfies as many clauses as possible.

   (a) Develop a linear programming relaxation for MAX-SAT. The relaxation should have the following variables: x_i for $i \in V$ should represent whether variable i is true, and z_c for $c \in \mathcal{C}$ should represent whether clause c is satisfied.

   (b) Suppose that we round a solution $\{x, z\}$ to the LP relaxation by setting each variable i to be true independently with probability x_i. Determine a lower bound on the probability that a clause c is satisfied; relate this probability lower bound to z_c.

   (c) Use your answer to parts (a) and (b) to develop an approximation algorithm for MAX-SAT. Try to obtain the best approximation factor you can.