CS880: Algorithmic Mechanism Design
HW2

Due: Nov 2, 2020

Submission instructions: Please email a PDF to Shuchi by Nov 2, 11:59 pm.

Problems:

1. (10 points) Consider a revenue maximization setting with one item and n buyers, where for $i \in [n]$ the value of buyer i (denoted v_i) is drawn independently from regular distribution F_i. Let M denote the optimal (Myerson's) revenue achievable by the seller in this setting.

   Consider another setting with one item and $2n$ buyers, where the values of buyers i and $n+i$ for $i \in [n]$ are drawn independently from the distribution F_i. Let V denote the revenue of the Vickrey auction for this setting.

   (a) Provide an example where V is larger than M.

   (b) Provide an example where V is smaller than M.

   (c) Prove that for any tuple of regular distributions $F_1 \times \cdots \times F_n$ we have $V \geq M/2$.

   (d) Consider a mechanism for the first setting that draws an independent sample s_i from each of the distributions F_i and runs a Vickrey auction with the anonymous reserve price $\max_i s_i$. What can you say about the revenue of this mechanism relative to M? (Hint: relate this revenue to V.)

2. (5 points) Optimal mechanisms for symmetric settings[1] are always symmetric owing to the convexity of the space of all IC mechanisms. However, optimal pricings are not necessarily symmetric. Even in a 2-item setting with a single unit-demand agent whose values for the items are drawn from an independent symmetric distribution, there may be no symmetric optimal item pricing. Identify a single dimensional distribution F for which there is no symmetric optimal item pricing for a two item setting with a unit-demand agent whose values for each of the items are drawn i.i.d. from F.

3. (5 points) Let $F_1, \cdots, F_m$ be independent value distributions, and for a set $S \subset [m]$ let F_S denote the distribution over value vectors where the value of an item $i \in S$ is drawn independently from F_i and the value of an item $i \notin S$ is 0.

   Let $\mathrm{Rev}(F_S)$ denote the optimal revenue achievable from an additive buyer with item values drawn from F_S. Let $\mathrm{SRev} = \sum_{i \in [m]} \mathrm{Rev}(F_{\{i\}})$.

   Prove that there exists a constant $\alpha > 0$ such that for any distribution $q = \{q_S\}_{S \subseteq [m]}$ over subsets of $[m]$, we have:

$$\sum_{S \subseteq [m]} q_S \,\mathrm{Rev}(F_S) \leq \alpha \cdot \mathrm{E}_{S \sim q}[|S|] \cdot \mathrm{SRev}$$

[1] i.e., where the distribution and the feasibility constraint are symmetric across items and/or buyers.