CS880: Algorithmic Mechanism Design
HW2

Due: Nov 2, 2020

Submission instructions: Please email a PDF to Shuchi by Nov 2, 11:59 pm.

Problems:

1. (10 points) Consider a revenue maximization setting with one item and \( n \) buyers, where for \( i \in [n] \) the value of buyer \( i \) (denoted \( v_i \)) is drawn independently from regular distribution \( F_i \). Let \( M \) denote the optimal (Myerson’s) revenue achievable by the seller in this setting.

Consider another setting with one item and \( 2n \) buyers, where the values of buyers \( i \) and \( n+i \) for \( i \in [n] \) are drawn independently from the distribution \( F_i \). Let \( V \) denote the revenue of the Vickrey auction for this setting.

(a) Provide an example where \( V \) is larger than \( M \).
(b) Provide an example where \( V \) is smaller than \( M \).
(c) Prove that for any tuple of regular distributions \( F_1 \times \cdots \times F_n \) we have \( V \geq M/2 \).
(d) Consider a mechanism for the first setting that draws an independent sample \( s_i \) from each of the distributions \( F_i \) and runs a Vickrey auction with the anonymous reserve price \( \max_i s_i \). What can you say about the revenue of this mechanism relative to \( M \)? (Hint: relate this revenue to \( V \).)

2. (5 points) Optimal mechanisms for symmetric settings\(^1\) are always symmetric owing to the convexity of the space of all IC mechanisms. However, optimal pricings are not necessarily symmetric. Even in a 2-item setting with a single unit-demand agent whose values for the items are drawn from an independent symmetric distribution, there may be no symmetric optimal item pricing. Identify a single dimensional distribution \( F \) for which there is no symmetric optimal item pricing for a two item setting with a unit-demand agent whose values for each of the items are drawn i.i.d. from \( F \).

3. (5 points) Let \( F_1, \cdots, F_m \) be independent value distributions, and for a set \( S \subset [m] \) let \( F_S \) denote the distribution over value vectors where the value of an item \( i \in S \) is drawn independently from \( F_i \) and the value of an item \( i \not\in S \) is 0.

Let \( \operatorname{Rev}(F_S) \) denote the optimal revenue achievable from an additive buyer with item values drawn from \( F_S \). Let \( \operatorname{SRev} = \sum_{i \in [m]} \operatorname{Rev}(F_{\{i\}}) \).

Prove that there exists a constant \( \alpha > 0 \) such that for any distribution \( q = \{q_S\}_{S \subseteq [m]} \) over subsets of \([m]\), we have:

\[
\sum_{S \subseteq [m]} q_S \operatorname{Rev}(F_S) \leq \alpha \cdot \mathbb{E}_{S \sim q}[|S|] \cdot \operatorname{SRev}
\]

\(^1\)i.e., where the distribution and the feasibility constraint are symmetric across items and/or buyers.