CS 880 : Algorithmic Mechanism Design.

Prof. Shuchi Chawla

[Pswd for Zoom meetings : vickreycg]

Prerequs : CS 577 ; CS 787.

[Approx
Randomness
LP, Duality]

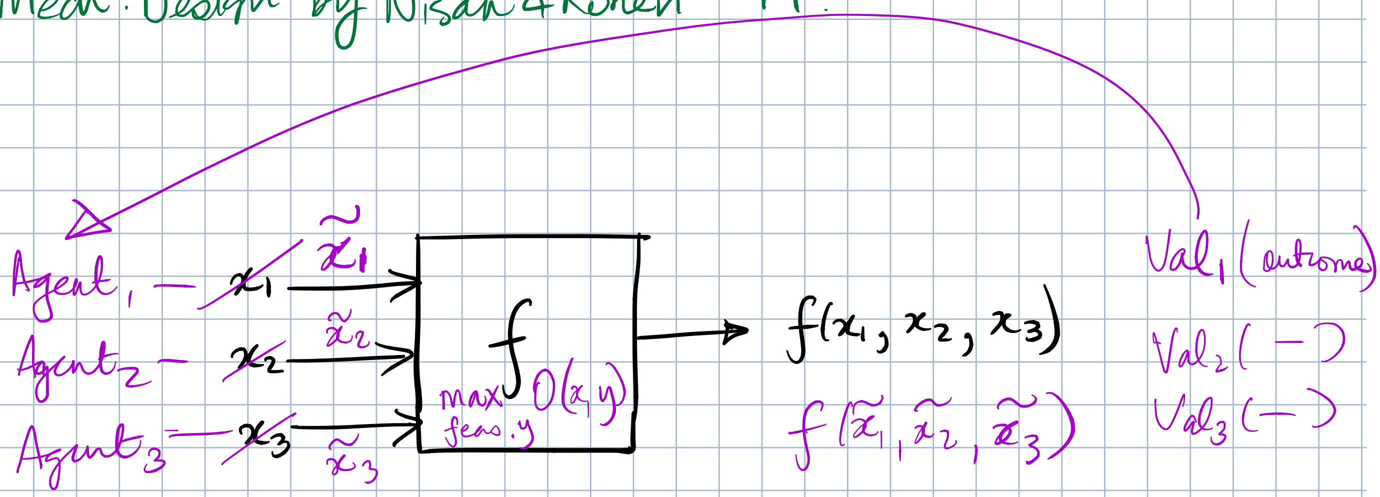
Expected work :

- 4 HWs.
- Everyone either grades a HW
  or scribes a lecture (1 week - 2.5 hrs)
- Indep. reading / research project
  - Presentation in class + project report

Mechanism Design = Optimization + Strategic behavior

(Optimization: Alg. Design)

Alg. Mech. Design by Nisan & Ronen '99.



$x \longrightarrow \tilde{x}$

$f(x) \longleftrightarrow f(\tilde{x})$

mech. design without money

Also, social choice theory

Question 1: How to incentivize agents to report input truthfully?

— By using money or payments

Question 2: How to predict agents' behavior?

Does strategic behavior hurt overall performance?

QUASILINEAR UTILITY

— We will study non-linear modes too.

Agents maximize util(outcome, payment)

$=$ val(outcome) $-$ amount they pay.

"QUASI" ... LINEARITY

Why Algorithmic Mechanism Design?

- Modern markets are increasingly large scale / algorithmic in nature
- Distributed use of computational resources requires economic approaches.

Objectives

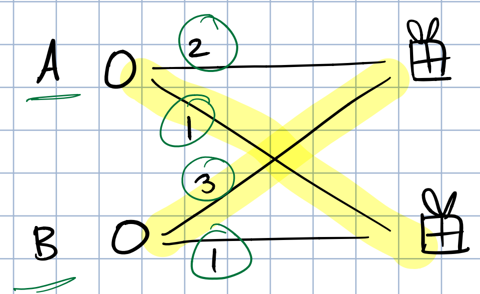
- SOCIAL WELFARE or ECONOMIC EFFIENCY

  resources go to those that benefit from ~~value~~ them the most.

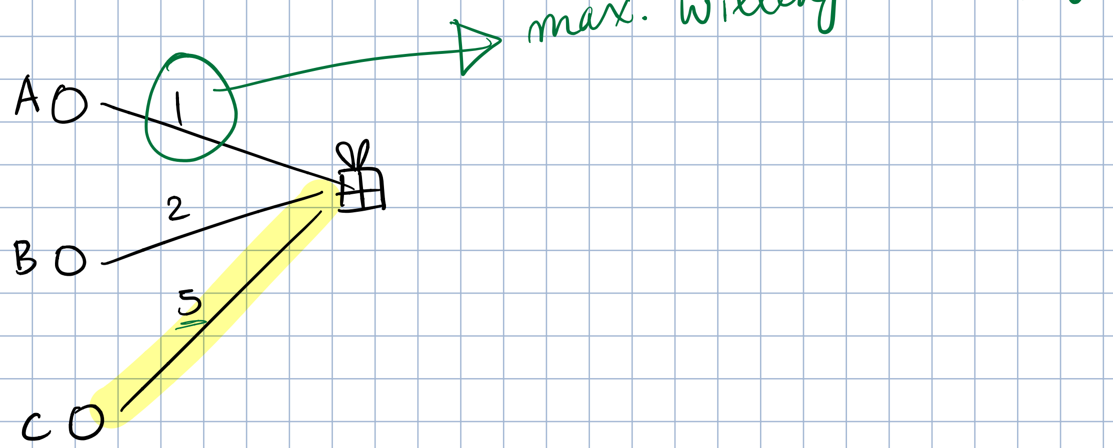
- REVENUE

Other objectives e.g. fairness, makespan.

Example : maximum bipartite / matching — can find efficiently



Simpler example :



Some approaches.

- First price sealed-bid auction — Not truthful. Bid amts depend on agents' beliefs about others.

- English auction (Ascending auction) Always efficient

- Dutch auction (Descending auction ; clock auction)

  Identical to FPA

- Second price sealed-bid auction (VICKREY auction)

  Highest bid wins

  Pays second highest bid

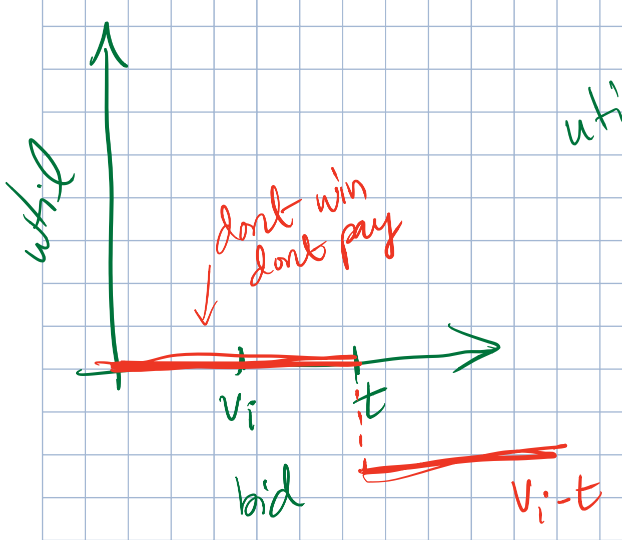
Claim: SPA is "truthful" Dominant Strategy Incentive Compatible. (DSIC)

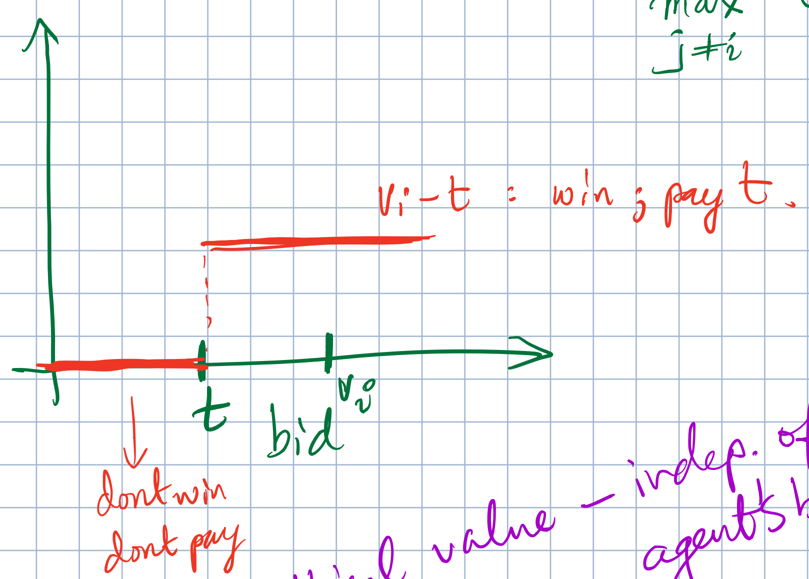
Incentive Compatible : Agents report true values.

Dominant Strategy : No matter what others do/believe. (DS Equilibrium)

critical value

Proof: Fix agent i. Value v_i. Denote by $t = \max_{j \neq i} b_j$





Critical value – indep. of agent's bid

⇓

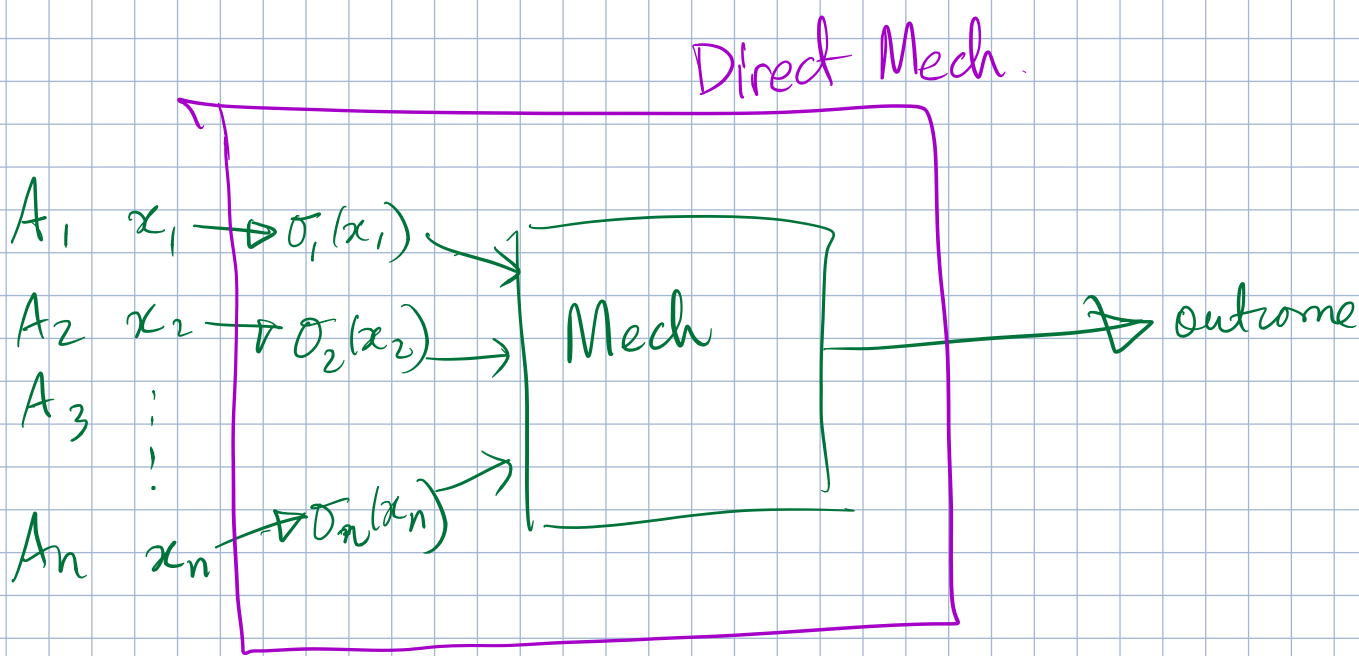
DSIC

DIRECT vs INDIRECT MECHANISMS.

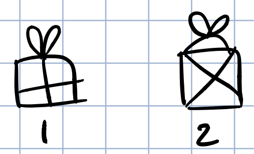
- Indirect → protocols with many steps.
- Direct → Ask agents to report values.

REVELATION PRINCIPLE

If a social choice function can be implemented by an arbitrary mechanism in a certain equilibrium, then it can be implemented by a direct incentive-compatible mechanism in the same equilibrium concept.



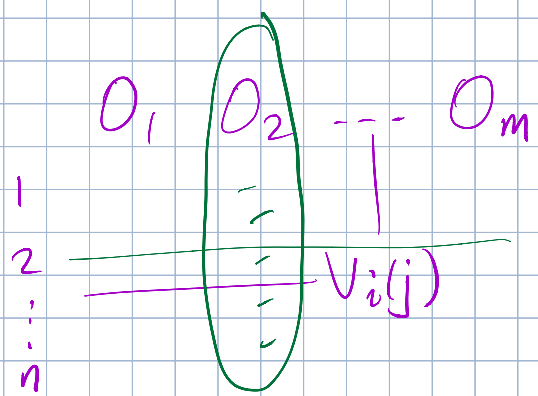
Combinatorial Auctions.



Value function maps subsets of items to values.

$V_A(\{1\}) = 3$ $\quad V_A(\{2\}) = 3$ $\quad V_A(\{1,2\}) = 3$

$V_B(\{1\}) = 2$ $\quad V_B(\{2\}) = 2$ $\quad V_B(\{1,2\}) = 4$



corresponds to an allocation

More generally: n agents, m possible outcomes $O_1, \ldots O_m$.

Valuation functions $V_i(j)$ for $i \in [n]$, $j \in [m]$.

Goal: $\max_{j \in [m]} \sum_{i \in [n]} V_i(j)$ SOCIAL WELFARE

Vickrey-Clark-Groves mechanism. (VCG)

- Ask agents to report value functions.
- Compute $j = \operatorname{argmax} \sum_i V_i(j)$
- Charge each agent their "critical value".

Claim: VCG is DSIC mechanism

Proof:

	O_1	O_2	$\cdots$	O_m
i	$v_i(1)$	$v_i(2)$	$\cdots$	$v_i(m)$
"$-i$"	$v_{-i}(1)$	$v_{-i}(2)$	$\cdots$	$v_{-i}(m)$

	O_1	O_2
i	4	1
$-i$	4	6
	2	0

$$v_{-i}(j) = \sum_{i' \neq i} v_{i'}(j)$$

$$p_j = \max_{j'} v_{-i}(j') - v_{-i}(j)$$

p_j → critical prices

Agent i wants to maximize $v_i(j) - p_j$

$$\arg\max_j \{v_i(j) - p_j\} = \arg\max_j \left\{ v_i(j) - \left\{ \max_{j'} v_{-i}(j') - v_{-i}(j) \right\} \right\}$$

$$= \arg\max_j SW(j) - \max_{j'} v_{-i}(j')$$

$$= \arg\max_j SW(j)$$

aligns agents' opt. prob. with auctioneer's objective

Properties of VCG

- Efficient
- Dominant Strategy Incentive Compatible. (DSIC).

Problems with VCG.

- Needs exact computation of optimum — often computationally hard.
- Needs too much communication.
- Revenue non-monotonicity.
- Collusion; false name bids.