Profit Guaranteeing Mechanisms for Multicast Networks

S. Chawla D. Kitchin U. Rajan R. Ravi A. Sinha
Carnegie Mellon University, Pittsburgh, PA, USA.
{shuchi@cs., dkitchin@andrew., urajan@andrew., ravi@, asinha@andrew.} cmu.edu

Categories and Subject Descriptors

F.2.0 [**Analysis of Algorithms**]: General

General Terms

Algorithms, Theory

Keywords

Mechanism Design, Profit Maximization, Multicast Game

1. INTRODUCTION

We consider the design of multicast networks when both edges and nodes are selfish agents with private values. Given a network with a distinguished node (root) and clients at other nodes, the problem is to select a set of users for service and construct a multicast tree T connecting these users to the root. Clients are willing to pay some maximum amount of money u_i for receiving the service and agents owning edges charge a minimum cost c_e for participating. The multicaster's task is to charge a fee $f_i \leq u_i$ from the clients selected to receive service, and pay agents owning the edges used, a fee $f_e \geq c_e$, so as to maximize profit, $\pi = \sum_i f_i - \sum_e f_e$, from the transaction. Our objective is to construct a polynomial time, strategy-proof mechanism, that achieves approximately the maximum achieveable profit.

The design of multicast networks in a game-theoretic setting has received a lot of attention recently. Prior to our work, researchers (see [4, 7] and references therein) have only studied a simpler problem, in which edge costs are known to the multicaster. The objectives for this problem have been two-fold: to recover the cost of constructing the network T from the nodes (*Budget balance*: $\sum_{i\in T} f_i \geq \sum_{e\in T} f_e$), and to maximize the surplus (*Efficiency* $\phi = \sum_{i\in T} u_i - \sum_{e\in T} c_e$). It is well known that the two objectives cannot be met simultaneously [7, 3]. In order to obtain an approximation to profit, a mechanism should be both budget balanced and approximately efficient.

EC'03, June 9–12, 2003, San Diego, California, USA.
ACM 1-58113-679-X/03/0006.

From an optimization perspective, this problem reduces to finding a subset of the graph such that the sum of values u_i of selected nodes minus the sum of costs c_e of selected edges (efficiency) is maximized. Approximating this to within any factor in polynomial time is known to be NP-hard [4]. However, the related problem of minimizing the cost of the selected edges and the values of the nodes *not selected* (Prize collecting Steiner tree, abbreviated PCST) can be approximated to within a factor of 2. This algorithm is due to Goemans and Williamson [6], and we use it in our mechanism, referring to it as GW.

2. MECHANISM AND ANALYSIS

2.1 Profit guaranteeing mechanisms

Not only is the optimization problem inapproximable to any factor, but it is also impossible to simultaneously achieve budget balance and even approximate efficiency. In view of these impossibility results, we introduce the following measure of how good a mechanism is when it comes to raising profit. For the rest of this paper, we assume that u_i actually refers to the maximum revenue that any strategy-proof mechanism can raise from node i. For a tree T, let $u(T) = \sum_{i\in T} u_i$ and $c(T) = \sum_{e\in T} c_e$. Let $U = \sum_i u_i$.

Definition 1. Let $\alpha \in [0, 1]$ and $\beta \geq 1$. A mechanism is called (α, β)-profit guaranteeing if it satisfies the following:

1. Strategyproofness, Consumer sovereignity, Individual rationality, and Polynomial time computability.
2. There exists a non-negative function $k(\delta)$, that is strictly increasing for $\delta > 1 - \alpha$ and approaches 1 as $\delta \to 1$, and for which the following holds: if the optimal efficiency is at least δU for $\delta > 1 - \alpha$, the mechanism finds a tree with profit at least $k(\delta)U$.
3. If $c(T) \geq \beta u(T)$ for every tree T, it demonstrates that no non-trivial positive surplus tree exists.
4. If neither of the conditions in 2 and 3 above are met, the mechanism returns a solution with non-negative profit (possibly the trivial solution, with no node served and no edges selected).

Thus profit-guaranteeing mechanisms guarantee to generate a constant fraction of the best achievable profit if a highly profitable solution exists, and provide a proof if all solutions are highly unprofitable. By definition, if the mechanism returns a non-trivial solution, the solution has positive efficiency and satisfies budget balance.

2.2 Our mechanism

In addition to the GW algorithm for PCST, we make use of some other previous work to construct a profit-guaranteeing mechanism. Bikhchandani et al [1] give a strategyproof mechanism based on the Vickrey auction for constructing a Minimum Spanning Tree when edges are selfish agents. We note that their mechanism remains strategyproof even if a subset of the MST is selected, so long as the selection is based on Vickrey prices of edges and not their true costs. We use the mechanism of Bikhchandani et al to determine payments for edges.

We determine payments for nodes using a cancellable auction due to Fiat et al [5]. An auction is cancellable if the auctioneer has the option of cancelling the auction based on some pre-specified criterion, and this does not affect the strategies of the participants.

In order to achieve a profit guarantee, we require that there are at least two clients at every node, so that we can run cancellable auctions at nodes. We also require that for every edge cut in G, the ratio between the costs of the minimum and second minimum cost edges is at most a small constant, say $(1+\epsilon)$. A graph satisfying these properties is called ϵ-competitive. It is easy to construct examples to show that these conditions are necessary to obtain any profit guarantee [2]. We discuss the relaxations of these assumptions in the next section. Our mechanism is described below:

Mechanism M

1. Run a cancellable auction at every node in the graph. Use the value of the revenue obtained as the "revealed" node utility. Ask edges to reveal their bids.
2. Use GW to approximately solve PCST using the revealed edge and node utilities. Let V' be the set of nodes selected for service, and define $G' = (V', E')$ to be the subgraph of G induced by V'.
3. Construct a minimum spanning tree T_1 on G'. The edges in T_1 are paid their Vickrey prices. Prune the solution from bottom up to improve its efficiency based on Vickrey prices on edges:
   (a) For each node i, let $e(i)$ denote its parent edge in the MST and $ch(i)$ its children nodes.
   (b) Compute the *surplus* $\pi(i)$ of each node as follows. For a leaf node, $\pi(i) = f_i - f_{e(i)}$, and for an internal node, $\pi(i) = f_i - f_{e(i)} + \sum_{j \in ch(i):\pi(j)>0} \pi(j)$.
   (c) Identify all nodes with negative surplus. Delete the subtrees rooted at these nodes. Call this pruned solution T.
4. If the GW tree T_1 is non trivial, return the solution T.
5. If T_1 is trivial, rescale all the node utilities to $u'_i = 2u_i$ and rerun GW. If the algorithm again returns the trivial solution, output "Fail, no positive solution exists".

We obtain the following profit guarantee:

THEOREM 1. *Mechanism M is a $(\frac{1}{8(1+\epsilon)}, 4)$-profit guaranteeing mechanism, with $k(\delta) = 1 - 8(1+\epsilon)(1-\delta)$, on an ϵ-competitive graph.*

The proof of this theorem is omitted from this abstract and appears in the full version of this paper [2]. We also note that the efficiency of the solution produced by Mechanism M is at least $(2\delta - 1)U$ whenever $\delta > 1 - \alpha$.

2.3 Relaxation of assumptions

Suppose we relax the assumption that there be at least two clients at every node. In the simplest case with just one edge between the client and the root, this becomes the bilateral trading problem without any (distributional or range) assumptions on the inputs. It is impossible to devise a mechanism with any guarantee in this situation.[1] Mechanism M is trivially a $(0, 4)$-profit guaranteeing for this case.

However, if the node values u_i are known to the multicaster, we can relax the assumption of at least two clients at every node. In this case, we use u_i as the revealed node utilities in Step 1 of Mechanism M, and continue as before. This results in a $(\frac{1}{2(1+\epsilon)}, 4)$-profit guaranteeing mechanism.

Any mechanism for our general problem must pay Vickrey prices to edges. In the absence of ϵ-competitiveness among edges, the Vickrey prices could be arbitrarily high, eliminating any hope for good mechanisms. Essentially, mechanisms with no assumptions on the distribution of values of the players can achieve guarantees only by playing them off against one another, necessitating some kind of competitiveness in order to provide guarantees on their performance.

3. OPEN QUESTIONS

Improving the parameters of our profit guaranteeing mechanism remains open. It would also be interesting to see if such mechanisms exist for other hard problems. Another interesting problem would be to consider the situation in which agents own subsets of edges rather than individual edges. Our mechanism does not seem to extend to this case.

We thank Avrim Blum and Christine Parlour for several useful discussions.

4. REFERENCES

[1] S. Bikhchandani, S. de Vries, J. Schummer, and R. Vohra. Linear programming and Vickrey auctions. *Mathematics of the Internet: E-auction and Markets, IMA Volume on Mathematics and its Applications*, 127:75–116, 2002.

[2] S. Chawla, D. Kitchin, U. Rajan, R. Ravi, and A. Sinha. Profit maximizing mechanisms for the extended multicasting game. *CMU CS Technical report CMU-CS-02-164*, 2002.

[3] J. Feigenbaum, A. Krishnamurthy, R. Sami, and S. Shenker. Hardness results for multicast cost sharing. In *Proc. Conf. Foundations of Software Tech. and Theoretical Computer Science*, pages 133–144, 2002.

[4] J. Feigenbaum, C. Papadimitriou, and S. Shenker. Sharing the cost of multicast transmissions. *J. Computer and System Sciences*, 63:21–41, 2001.

[5] A. Fiat, A. Goldberg, J. Hartline, and A. Karlin. Competitive generalized auctions. In *Proc. 34^{th} ACM Symp. on Theory of Computing*, pages 72–81, 2002.

[6] M. Goemans and D. Williamson. A general approximation technique for constrained forest problems. *SIAM J. Comput.*, 24(2):296–317, 1995.

[7] H. Moulin and S. Shenker. Strategyproof sharing of submodular costs: budget balance vs efficiency. *Economic Theory*, 18:511–533, 2001.

[1]Note that for the bilateral trading problem with one buyer and one seller, in the presence of a distributional assumption, there exists a simple Incentive Compatible mechanism that achieves a constant fraction of the achievable profit in expectation. Likewise, given a range for the buyer and seller bids, one can construct a mechanism that obtains a constant fraction of the profit if the maximum achieveable profit is sufficiently high.