CS412 Spring Semester 2011

Homework Assignment #6

Due Thursday May 5th 2011, in class

Sum of all problems : 120%, Maximum possible score : 100%.

1. [40%] In addition to the Forward Euler, Backward Euler and Trapezoidal Rule methods, another simple methodology for approximating the solution to an initial value problem $y' = f(t, y)$ is the *modified Euler method*:

$$y_{k+1} = y_k + \Delta t f(\frac{t_k + t_{k+1}}{2}, \frac{y_k + y_{k+1}}{2})$$

   (a) [30%] Examine under what conditions this method is stable, when applied to the model ODE $y'(t) = \lambda y(t),\ \lambda < 0$.

   (b) [10%] In some cases, this method can be quite similar to the Trapezoidal Rule. Describe one ODE where modified Euler and Trapezoidal rule reduce to the same method, and one more ODE where they produce different results.

2. [30%] For the following ordinary differential equations, determine whether their solutions are stable, asymptotically stable, or unstable:

   (a) $y'(t) = \sin(2y(t))$ with $t \geq 0$.

   (b) $y'(t) = \int_0^t e^{-\tau^2} d\tau$ with $t \geq 0$.

   (c) $y'(t) = -ye^t$ with $t \geq 0$.

3. [50%] Consider the definite integral:

$$I = \int_{-1}^{1} \sqrt{1 - x^2} dx$$

   The function $f(x) = \sqrt{1 - x^2}$ being integrated is simply the half-circle centered at the origin, with a radius of one, and positive y coordinates. Thus the exact value of this integral is the area of a half unit disc, i.e. $I = \pi/2$.

   Use `MATLAB` to approximate this integral using either

   (a) The composite trapezoidal rule

   (b) Composite Simpson's rule.

   Use each of the two methods to approximate this integral using 5, 9 or 17 equidistant data points. Compute the value of the error from the exact value $I = \pi/2$, for each individual method, and each number of data points. **Note:** You should implement these two composite integration methods from scratch, not emulate them using built-in `MABLAT` functions.