CS412 Spring Semester 2012

Homework Assignment #1

Due Tuesday February 21st 2012, in class

Sum of all problems : 120%, Maximum possible score : 100%.

For the following problems, even if you do not answer a given question, you are still allowed to use its result in order to answer a subsequent question.

1. [45%] Consider the following pairs, each containing one nonlinear equation and one iterative procedure:

   (a) The equation $e^x = x + 2$ and the iterative method
   $$x_{k+1} = e^{x_k} - 2$$

   (b) The equation $x^3 = x^2 + 1$ and the iterative method
   $$x_{k+1} = \frac{x_k}{1 + x_k^2}$$

   (c) The equation $3\theta = \cos 2\theta$ and the iterative method
   $$\theta_{k+1} = \frac{\theta_k + \cos 2\theta_k}{4}$$

   For each case, examine if the given iterative procedure is an effective solution technique for the respective equation. In order for the method to be effective it needs to (a) be guaranteed to converge and (b) converge to a solution of the given equation.

2. [30%] Use Newton's method to generate an iterative process that converges to the following values:

   (a) Create an iterative procedure that computes the cube root $\sqrt[3]{a}$ of a given number a. You are **not allowed** to use roots in the formula. [Hint: The cube root is the solution of $x^3 - a = 0$]

   (b) Create an iterative procedure that computes the natural logarithm $\log a$ of a given number a. You are **not allowed** to use logarithms in the formula (exponentials are ok).

   (c) Create an iterative procedure that computes the arc-tangent $\arctan(x)$ of a given number x (remember, the arc-tangent of x is the angle whose tangent equals x). You are **not allowed** to use inverse trigonometric functions in the formula (normal trigonometric functions such as $\sin, \cos, \tan$ are ok).

3. [45%] Consider the following procedure for solving the nonlinear equation $f(x) = 0$

- Start with an initial guess x_0
- For $k = 0, 1, 2, \ldots$ do the following:
  - Compute the value that the standard Newton method would provide, and call it $\hat{x}_{k+1}$, i.e.

$$\hat{x}_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

  - Compute the next approximation x_{k+1} by averaging x_k and $\hat{x}_{k+1}$, i.e.

$$x_{k+1} = \frac{x_k + \hat{x}_{k+1}}{2}$$

(a) [10%] Show that if this method converges, it will converge to a solution of $f(x) = 0$

(b) [15%] Show that this method converges under the same conditions as Newton's method.

(c) [15%] Determine the order of convergence of this method.

(d) [5%] Would we ever want to use this method, instead of Newton's?