

CS412 Spring Semester 2013

Homework Assignment #1

Due Thursday February 7th 2013, in class

Sum of all problems : 120%, Maximum possible score : 100%.

For the following problems, even if you do not answer a given question, you are still allowed to use its result in order to answer a subsequent question.

1. [30%] Examine if the function $g(x)$ is a contraction in the following cases

(a) $g(x) = \frac{2}{3} \sin(\frac{4x}{3}), x \in \mathbf{R}$

(b) $g(x) = 2\sqrt{x}, x \in [2, +\infty)$

(c) $g(x) = \frac{1}{2}e^{-x}, x \in \mathbf{R}$

2. [60%] Consider the following pairs, each containing one nonlinear equation and one iterative procedure:

- (a) The equation $e^x = x + 2$ and the iterative method

$$x_{k+1} = e^{x_k} - 2$$

- (b) The equation $x^3 = x^2 + 1$ and the iterative method

$$x_{k+1} = \frac{x_k}{1 + x_k^2}$$

- (c) The equation $3\theta = \cos 2\theta$ and the iterative method

$$\theta_{k+1} = \frac{\theta_k + \cos 2\theta_k}{4}$$

- (d) The equation $x^2 - a = 0$ and the iterative method

$$x_{k+1} = \frac{3x_k^2 + a}{4x_k}$$

For each case, examine if the given iterative procedure is an effective solution technique for the respective equation. In order for the method to be effective it needs to (a) be guaranteed to converge and (b) converge to a solution of the given equation.

3. [30%] Use Newton's method to generate an iterative process that converges to the following values:

- (a) Create an iterative procedure that computes the power $a^{\frac{4}{3}}$ of a given number a . You are **not allowed** to use roots, or raise numbers to fractional powers in the formula. [Hint: This value is the solution of $x^3 - a^4 = 0$]

- (b) Create an iterative procedure that computes the arc-sine $\arcsin(x)$ of a given number $x \in [0, 1]$ (remember, the arc-sine of x is the angle whose sine equals x). You are **not allowed** to use inverse trigonometric functions in the formula (normal trigonometric functions such as sin, cos, tan are ok).