Spectral graph wavelet is constructed by applying a band-pass filter to the wavelet basis function, localized in both time and frequency. Similar to the Fourier transform, but uses wavelet basis function instead of a continuous function. The mother wavelet is formed by interchanging the roles of position and frequency, caused by infinite duration of basis function in Fourier transform. In order to filter the network structure, we need to bring the network connectivity information as a signal into another domain.

**WAVELET IN THE NON-EUCLIDEAN SPACE (CONTINUED)**

- The wavelet transform of a given function \( f \) is derived from graph Fourier transform and yields the wavelet coefficients.
  
  \[
  W_l(s,n) = \langle \psi_s, f \rangle = \sum_{n \in V} \sum_{j=0}^{N-1} g(\alpha_s) \hat{f}(j) \chi_j(n)
  \]

- Wavelet Multiscale Descriptor (WMD): multi-resolutional shape descriptor

\[
WMD_l(n) = \langle W_l(s,n) | s \in S \rangle
\]

**LINE GRAPH (DUAL GRAPH) TRANSFORMATION**

- Line graph \( L(G) \) is a dual form of graph \( G \).
- The \( L(G) \) is formed by interchanging the roles of \( V \) and \( E \) in \( G \).
- When two edges share a common vertex in \( G \), these edges are connected to each other by the common vertex.
- Let \( e \) be the elements in the adjacency matrix \( A \) of \( L(G) \), then
  
  \[
  e = \begin{cases} 
  1 & \text{if } v \in V, v - v' \in E \\
  0 & \text{otherwise}
  \end{cases}
  \]

**NETWORK FILTERING PROCESS**

- In order to filter the network structure, we need to bring the network connectivity information as a signal into another domain.
- Using line graph, we transform our domain \( G \) to \( L(G) \), where edge weights are viewed as a signal defined on vertices.

**DATASET**

- 140 subjects (41 male, 100 female) in Wisconsin registry for Alzheimer’s prevention (WRAP) cohort.
- Brain connectivity strengths between 162 gray matter regions from Destrieux atlas were defined as the mean fractional anisotropy (FA) along the tracts of diffusion tensor imaging.
- 5 different scales of the wavelet based multi-resolution connectivity signatures (WaCS) for each connectivity.
- Pittsburgh compound B (PiB) distribution volume ratios (DVR) of 16 different regions.

**EXPERIMENTAL RESULT**

- The false discovery rate (FDR) threshold curve is also shown (red).
- The \( p \)-values for WaCS (blue) for 10 out of the 16 ROIs clearly show the advantage of having multi-resolutional views.
- For 7 out of these 10 ROIs the statistical significance is at the FDR corrected level (blue above red).
- Forcing regions in the entire brain, the multiple resolutions and individual scales seem to perform similarly at the uncorrected level.

**CONCLUSION**

- We presented a unique multi-resolutional statistical analysis to study the influence of amyloid burden on structural brain connectivity.
- In almost all the regions implicated as important in Alzheimer’s disease the multiple resolutions tend to improve the distribution of the \( p \)-values.