DISCUSSION 10

-Review on fractional factorial:

1. compounding, alias

   Columns that have identical entries correspond to effects that are **confounded** or **aliased**. In order to construct a fractional factorial that deliberately confounds pre-selected important factors, one needs to use a generator (=**word**). The generator uses the fact that squaring the entries in any given column gives a column of ones, which can be thought of as an identity element I.

2. defining relation, generators, resolution,

   defining relation is the whole set of all the generators:

   D = AB, I =ABD(generators in the standardized form)

   NOTE: I = -ABC can also work as a generator ,i.e. allow up to sign difference.

   e.g. 2^{7-4}_{III} design, with D = AB, E=AC, F= BC, G=ABC.

   # independent generators = ? , # of words in the defining relation= ?.

3. higher-order interaction negligible:

   Large factorial designs allow for estimation of many higher order effects. If we can make assumptions that three-factor or higher-order interactions are negligible. Fractional factorial designs employ such redundancy by arranging that lower order effects are compounded with those of higher order.

4. parsimony, projectivity $P = R - 1$

   the vital few factors and trivial many – Pareto Principle / Occam's Razor

   projectivity = P: Every subsets of P or fewer factors produce a complete factorial (possibly replicated). f the design project on them

5. sign switching, fold over, and sequential assembly

   Further runs may be needed when fractional designs yield ambiguities. One strategy in this sequential assembly of designs is called "**foldover**". It is achieved by signswitching. Increase design with 8 runs to a design with 16 runs.

   (a) single-column fold over
   (b) multiple-column fold over
   (c) fold over the entire design
   (d) How to determine the generators/defining relations in the combined design ?

-Review on Plackett and Burman Design & boxcox transformation:

1. comparison with fractional factorial design

(a) in fractional factorial the number of runs n has to 4,8,16,32,..., but the gaps become wider as n increases.

(b) PB design is another type of two-level orthogonal design. these designs are available for any n that is a multiple of 4, in particular 12, 20, 24, 28 .... An arrangement of this kind is also called orthogonal array and is denoted as L_n

(c) L_{12} PB design for 5 factors, its alias table:

Table 7.4. Alias Structure, Twelve-Run PB Design Employing Five Factors

Contrasts		Alias Structure
l_A	$\rightarrow$	$\mathbf{A} + \frac{1}{3}(-BC + BD + BE - CD - CE - DE)$
l_B	$\rightarrow$	$\mathbf{B} + \frac{1}{3}(-AC + AD + AE - CD + CE - DE)$
l_C	$\rightarrow$	$\mathbf{C} + \frac{1}{3}(-AB - AD - AE - BD + BE - DE)$
l_D	$\rightarrow$	$\mathbf{D} + \frac{1}{3}(+AB - AC - AE - BC - BE - CE)$
l_E	$\rightarrow$	$\mathbf{E} + \frac{1}{3}(+AB - AC - AD + BC - BD - CD)$
l_F	$\rightarrow$	$\frac{1}{3}(-AB + AC - AD + AE + BC - BD - BE + CD - CE - DE)$
l_G	$\rightarrow$	$\frac{1}{3}(-AB - AC - AD + AE - BC + BD - BE + CD - CE + DE)$
l_H	$\rightarrow$	$\frac{1}{3}(+AB + AC - AD - AE - BC - BD - BE - CD + CE + DE)$
l_J	$\rightarrow$	$\frac{1}{3}(-AB - AC - AD - AE + BC + BD - BE - CD - CE - DE)$
l_K	$\rightarrow$	$\frac{1}{3}(-AB - AC + AD - AE - BC - BD - BE + CD + CE - DE)$
l_L	$\rightarrow$	$\frac{1}{3}(-AB + AC + AD - AE - BC - BD + BE - CD - CE + DE)$

2. boxcox transformation

$$Y_\lambda = \begin{cases} \log(Y), & \lambda = 0 \\ \frac{Y^\lambda - 1}{\lambda}, & \lambda \neq 0. \end{cases}$$

λ-plot is called by function **boxCox{car}** in R, with log-likelihood as default evaluation criteria.

In class, we also use F-value as the criteria to draw the λ-plot. $(\lambda_i, F_i)_{i=1}^{100}$, then draw a plot.

3. Diagnostics of the data. There are many works that can be done

(a) look at the residual plot, any pattern, variance change over fitted values?

(b) look at the QQ-plot of residuals

(c) look at interaction plot. in R , use interaction.plot(x.factor, y.factor, response)

(d) $\cdots$

-Exercies:

1. Questions from previous homework and projects

(a) In project, How to determine the generators/defining relations in the combined design? further to determine the alias table?

(b) In project, how to conduct a linearity t-test?

(c) In homework, D=-AB, E= -ABC can work as a generators. write a set of generators? uniqueness of this set?

2. Question 7.1, page 315

1. Consider the following two-level experimental design for seven factors in 12 runs:

P	Q	R	S	T	U	V	Observed
+	−	−	−	+	+	+	37
+	+	−	+	+	−	+	23
+	−	+	+	−	+	−	40
−	+	−	−	−	+	+	37
−	+	+	+	−	+	+	5
−	−	−	−	−	−	−	39
+	+	−	+	−	−	−	7
+	−	+	−	−	−	+	44
−	−	+	+	+	−	+	51
+	+	+	−	+	+	−	35
−	−	−	+	+	+	−	56
−	+	+	−	+	−	−	33

Illustrate that (a) the design is orthogonal, (b) the design is of projectivity 3, and (c) the seven columns are those of a PB design.

potential danger : Its alias table is more complicated and thus the diagnostic methods for fractional factorial will not work for PB design.

e.g. The main effects of each of the five factors is alias to a string containing all the two-factor interaction not identified with that factor. interaction A&B will bias the remaining main effects and appearing in other interaction strings.

3. Data transformations, code in R on my homepage : www.stat.wisc.edu/∼songwang