Details for Split Plot Designs

Whole Plots Randomized as a CRD

Model

$$Y_{ijk} = \mu + \alpha_i + \varepsilon_{ij} + \tau_k + (\alpha\tau)_{ik} + \delta_{ijk}$$

where $i = 1, \ldots, a$; $j = 1, \ldots, b$ indexes whole plots for each whole plot treatment level (i.e. levels of A); $k = 1, \ldots, c$ indexes the subplot treatment levels (i.e. levels of C); $\varepsilon_{ij} \sim N(0, \sigma_\varepsilon^2)$ represents whole plot error; and $\delta_{ijk} \sim N(0, \sigma_\delta^2)$ represents subplot error.

ANOVA

Source	df	SS	MS	EMS
A	$a-1$	SSA	MSA	$\sigma_\delta^2 + c\sigma_\varepsilon^2 + \frac{bc\sum_i \alpha_i^2}{a-1}$
Whole Plot Error	$a(b-1)$	SSWPErr	MSWPErr	$\sigma_\delta^2 + c\sigma_\varepsilon^2$
C	$c-1$	SSC	MSC	$\sigma_\delta^2 + \frac{ab\sum_k \tau_k^2}{c-1}$
AC	$(a-1)(c-1)$	SSAC	MSAC	$\sigma_\delta^2 + \frac{b\sum_i\sum_k (\alpha\tau)_{ik}^2}{(a-1)(c-1)}$
SubPlot Error	$a(b-1)(c-1)$	SSSPErr	MSSPErr	σ_δ^2
Total	$abc-1$	SSTot		

The SubPlot Error is most easily calculated by subtraction. The WPError is equivalent to the SS for Whole Plots nested in A. In that sense, the Whole Plot Error reminds you of the sample or plot error in a design with subsampling. All other SS are calculated the usual way.

Tests

1. To test $H_0 : (\alpha\tau)_{ik} = 0$ for all i and k, we use:

$$F = \frac{\text{MSAC}}{\text{MSSPE}}$$

2. To test $H_0 : \tau_k = 0$ for all k, we use:

$$F = \frac{\text{MSC}}{\text{MSSPE}}$$

3. To test $H_0 : \alpha_i = 0$ for all i, we use:

$$F = \frac{\text{MSA}}{\text{MSWPE}}$$

Almost always, tests involving Whole Plot Treatments are less powerful than tests involving SubPlot Treatments.

Means Comparisons

Comparisons within the same Whole Plot Treatment

$$\text{Var}(\bar{Y}_{1\cdot 1} - \bar{Y}_{1\cdot 2}) = \frac{2\sigma_\delta^2}{b}$$

Comparisons across Whole Plot Treatments

$$\text{Var}(\bar{Y}_{1\cdot 1} - \bar{Y}_{2\cdot 1}) = \frac{2(\sigma_\delta^2 + \sigma_\varepsilon^2)}{b}$$

Comparisons of Whole Plot Treatments Means

$$\text{Var}(\bar{Y}_{1\cdot\cdot} - \bar{Y}_{2\cdot\cdot}) = \frac{2(\sigma_\delta^2 + c\sigma_\varepsilon^2)}{bc}$$

Comparisons of SubPlot Treatments Means

$$\text{Var}(\bar{Y}_{\cdot\cdot 1} - \bar{Y}_{\cdot\cdot 2}) = \frac{2\sigma_\delta^2}{ab}$$

Expected Mean Squares

$$\text{E(MSSWPE)} = \sigma_\delta^2 + c\sigma_\varepsilon^2$$

$$\text{E(MSSPE)} = \sigma_\delta^2$$

Whole Plots Randomized as an RCBD

Model

$$Y_{ijk} = \mu + \alpha_i + \beta_j + \varepsilon_{ij} + \tau_k + (\alpha\tau)_{ik} + \delta_{ijk}$$

where $i = 1, \ldots, a$; $j = 1, \ldots, b$ indexes blocks; $k = 1, \ldots, c$ indexes the subplot treatment levels (i.e. levels of C); $\varepsilon_{ij} \sim N(0, \sigma_\varepsilon^2)$ represents whole plot error; and $\delta_{ijk} \sim N(0, \sigma_\delta^2)$ represents sub-plot error.

ANOVA

Source	df	SS	MS	EMS
Block	$b-1$	SSBlk	MSBlk	$\sigma_\delta^2 + c\sigma_\varepsilon^2 + \frac{ab\sum_j \beta_j^2}{b-1}$
A	$a-1$	SSA	MSA	$\sigma_\delta^2 + c\sigma_\varepsilon^2 + \frac{bc\sum_i \alpha_i^2}{a-1}$
WP Error	$(a-1)(b-1)$	SSWPErr	MSWPErr	$\sigma_\delta^2 + c\sigma_\varepsilon^2$
C	$c-1$	SSC	MSC	$\sigma_\delta^2 + \frac{ab\sum_k \tau_k^2}{c-1}$
AC	$(a-1)(c-1)$	SSAC	MSAC	$\sigma_\delta^2 + \frac{b\sum_i\sum_k (\alpha\tau)_{ik}^2}{(a-1)(c-1)}$
SP Error	$a(b-1)(c-1)$	SSSPErr	MSSPErr	σ_δ^2
Total	$abc-1$	SSTot		

- The SSWPError is identical to the SS for the A by Block interaction (which is always the case in a block design).
- The df for SPError can also be written as:

  $$a(b-1)(c-1) = (b-1)(c-1) + (a-1)(b-1)(c-1)$$

  This gives a hint of an alternative way to find the SSSPError.