AUTOMATIC MRI IMAGE REGISTRATION USING PHASE CORRELATION

Robin Kramer

Department of Computer Science,
University of Wisconsin, Madison, United States
E-mail: spring@cs.wisc.edu

ABSTRACT

A technique for automatic Magnetic Resonance Image(MRI) registration is based on the Fast Fourier Transfrorm. This Technique uses images in the spectral domain to compute the phase correlation. The phase correlation relates two images by the Fourier shift property, and can be used to register MRI images.

1. INTRODUCTION

Image registration is an important problem in the medical imaging domain. There are standard images which have been hand registred and averaged to create composite images. These images then can be used to as a standard to register other images to them.

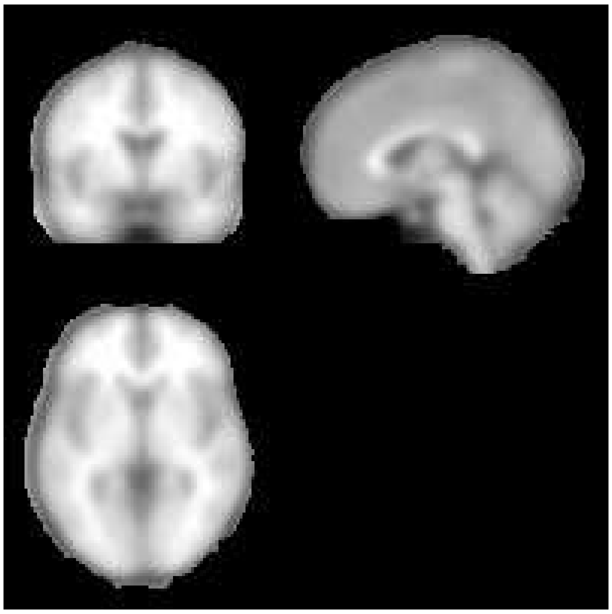

Figure 1: A Composite MRI volume

Originally, this data had a noise around the outside, called air speckles, and the tissue around the skull including the skull. The air speckles were removed using a despeckling program, and then the scalp was removed using the same program. Some of the brain tissue was probalble removed in addition to the scalp. To view, despecle, descalp and convert the images, a program called MRIcro was used. This program is freely available for unix and windows.

Studies, correlating activity among subjects, require the images be registered to one another. Making the images useful is a time consuming and computationally intensive task. MRI images are not easily reprsented, and therefore hand estimatimation of the angles and scales is difficult. Computationsally, the MRI data is very memory intensive and therefore is time consuming.

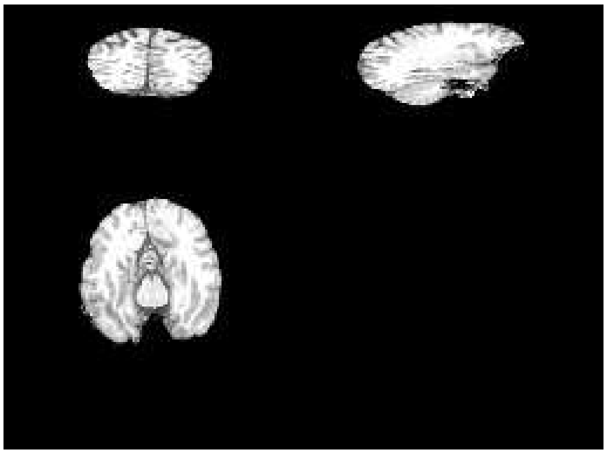

Figure 2: A single MRI scan of the brain

There is potential for applications in the medical imaging domain, one of which would be a MRI recognition program similar to eigen faces. A knee doctor who perfoms knee operations, could take an MRI image of the knee, which would then be projected into knee space. Then the most simialr knees could be presented to the doctor, who could then review them to make a decision about the patient.

To compare images for applications, these MRI images need to be registered. There have been successful methods for image registration. One of which was implemented by Oakes[4]. These methods have largely been based on detecting common features between images and computing a homography between them.

To register images which have large amounts of shift, which feature based warping does not handle well and is usually manipulated by a skilled operator before the registration algorithm is used, The Phase Correlation Method has been developed to take advantage of the Fourier shift property. Image registration algorithms based on Fast Fourier Transform(FFT) and Fourier Shift have existed for a number of years[1]. It is based on a well known property of continuous Fourier. Given I_1 is a shifted version of I_2, and F_1 and F_2 are the spectral images of I_1 and I_2 respectively, then these iamges are related by:

$$I_1(i,j,k) = I_2(i-a, j-b, k-c) \quad (1)$$
$$F_1(i,j,k) = F_2(i,j,k)e^{-j2\pi(i*a+j*b+k*c)} \quad (2)$$

Solving for the translation.

$$\frac{F_1(i,j,k)}{F_2(i,j,k)} = e^{-j2\pi(i*a+j*b+k*c)} \quad (3)$$
$$\frac{F_1(i,j,k)\times F_2^\star(i,j,k)}{F_2(i,j,k)\times F_2^\star(i,j,k)} = e^{-j2\pi(i*a+j*b+k*c)} \quad (4)$$

The $\star$ notation represents the complex conjugate. Multiplying by the conjugate is neccesary to calculate the ratio of the two images. The maximum of the inverse fourier of (4) is the the translation between the two images. An alternate method is to calculate.
$\frac{F_1(i,j,k)\times F_2^\star(i,j,k)}{|F_2(i,j,k)|\times|F_2(i,j,k)|}$
Where the $||$ is the $\sqrt{r^2+i^2}$ and r and i are the coefficients of the real and imaginary components. This is also called the phase.

The phase correlation method can only find how the same image can be translated to match another. To find differences in scaling and rotation between a target volume and a source volume, both the target and source volumes could be reparameterized, such that the result of the new parameterizations are volumes, in which translation could correspond to rotation and scaling in the original volumes/cite automatic. The phase correlation method can be used to solve for the translation between the two reparameterized volumes, and the source volume can then rotated and scaled to the target volume. Before the phase correlation method is used to solve for the translation for the original image.

The phase correlation method seems especially suited for use with MRI images. MRI images are intitially sampled in the spatial domain, and then later transformed in to the spatial domain. This means that they are bandwidth limited, and their should be no aliasing effects.

2. LOG-SPHERICAL COORDINATES

Logarithmic coordinates correspond to scaling in euclidean corrdinate system. The parameterized ordinate, which came from the logarithm of the euclidean distance $/sqrtx^2+y^2+z^2 = /rho$, translates to scaling about the orgin in the euclidean cordinate system. Spherical coordinates correspond to rotation in euclidean coordinates. The azimuth in spherical coordinates corresponds to rotation about the z axis in euclidean coordinates. Similarly, the altitude corresponded to rotation about the x axis. Both parameterizations can be performed simultaneously. The third ordinate in spherical cordinates is the distance from the origin. The phase correlation method can be used to solve for translation in the three dimensions of the reparameterized source and target volumes.

The base of the logarithm b was calculated from the length of one of the sides of the volume l. $b = 10^{/logl/l}$ The volumes must be cubes. The parameterization is

$$x = b^\rho \sin(\pi\phi/l) * \cos(2\pi\theta/l) \quad (5)$$
$$y = b^\rho \sin(\pi\phi/l) * \sin(2\pi\theta/l) \quad (6)$$
$$z = b^\rho \cos(\pi\phi/l) \quad (7)$$

To accomplish the reparameterization backwards mapping and tri-linear interpolation were used to ensure that there were no holes or measels.

Once the scale and rotation are known the images can be warped using a homogenous coordinate transformation with a 4 by 4 matrix. ϕ corresponding to rotation about the x axis. The θ corresponding to rotation about z axis.

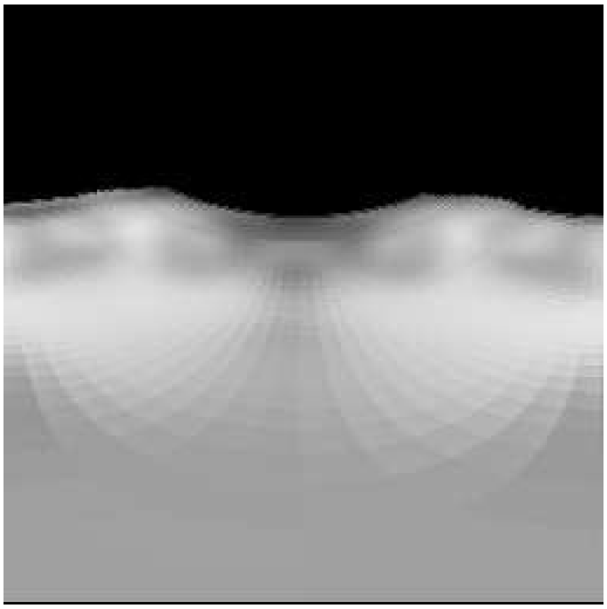

Figure 3: Log-Spherical Parameterized Coordinates

This result seems promising to Xie[1]. The property of the MRI image that was appealing, was the fact that image was bandwidth limited. The in converting the image to logpolar coordinates, the image is no longer bandwidth limited. One needs only look at the origin. At the origin all values for ρ and θ will have the same value. To have a unique sinusoid, there must be three points which are not colinear. These parameterizations are not bandwidth limited.

3. DISCRETE FOURIER TRANSFORM

The language that was used was C. Working with the volumes, was hindered by using virtual memory, but C allowed control of the allocation and dealloction of memory, so that the smallest amount of ram could be used at any one point in the program.

The Discrete Fourier was implemented from "Numerical Recipes in C". This algorithm calculated the N dimensional Fast Fourier Transform. This algorithm expected each dimension to be a power of two. Therefore the smallest cube with each side being a power of two was calculated, and the input data was placed in the center of the cubes which were initialized to zero.

The volumes were originally in an eight bit format, they were converted to a thirty-two bit floating point format, from which the FFT could be calculated. The volumes also had to be doubled in size to accomodate the storage of the imaginary components of the FFT. The intensities were then scaled and rounded back into eight bit values.

To test the phase correlation algorithm. An image was translated a known amount, then the FFT of the original and the new images were calculated. The phase ratio was calculated at each sinusoid in the FFTs. Then the inverse FFT was calculated. The location with the largest phase was chosen.

This yielded results that were similar to the original translations. For a known displacement of $x = 13$ $y = 10$ $z = 8$ voxels with an image which was $256 \times 256 \times 124$ voxels the phase correlation method yielded $x = 15$ $y = 47$ $z = 8$voxels. For a known translation of $x = 6$ $y = 15$ $z = 10$ voxels the phase correlation method yielded $x = 13$ $y = 52$ $z = 1$. For a known translation of $x = 16$ $y = 18$ $z = 39$ the phase correlation method yielded $x = 13$ $y = 52$ $z = 1$

4. EVALUATION

The algorithm is promising, even though there are some problems to be overcome. As with any numerical algorithm there is error from the functions not being continuous. The size of the representation is another limit on the performance of the matrix.

An interesting problem, is caused by the fact that pixel displacements of one pixel are represented by a a maximum in the correlation matrix exactly on the border. It is unfortunate, but FFT's are susceptible to edge noise and the inverse FFT is calculated on the correlation matrix. This would make it appear that the algorithm works least well translations which are small.

Another interesting problem, is that the phase ratio matrix only represents translations greater than zero. It infact does not even represent translations of images which are in the same axis.

5. CONCLUSION

Image registration is an important task in research an in medical applications. Currently most methods have limitations. The phase correlation method with parameterized coordinates promises to solve many of these problems, but currently has its own limitations. The key to these implementations as with most vision problems, is robustness. While results with the phase correlation method can be obtained under certain conditions, even some of the simpler problems are impossible to solve without extensions. The domain of medical imaging, stands to gain from solving these problems.

REFERENCES

[1] Hongjie Xie, Nigel Hicks, G.Randy Keller, Haitao Huang, Vladik Kreinovich. *Automatic Image Registration Based on A FFT Algorithm and IDL/ENVI.* In Pan American Center for Earth and Environmental Studies(Paces), Department of Geological Sciences University of Texas at El Paso, September 2000.

[2] William Scott Hoge. *A Subspace Identification Extension to the Phase Correlation Method.* .

[3] William H. Press, Saul A. Teukolsky, William T. Vetterling, Brian P. Flannery. *Numerical Recipes in C The Art of Scientific Computing Second Edition. Cambridge University Press*, Cambridge, UK, February 1993.

[4] Terry Oakes. *Coregistration.* In Lecture Notes/Interview, lecture(online http://tezpur.keck.waisman.wisc.edu/ oakes/teaching/Coregistration_l
.