

CS416 Spring 2007

Prof. Wright

Assignment #1

Due February 9, 2007

In this homework we examine loss of significance by evaluating the function $f(x) = \cos x - 1$ computationally for small x in two different ways: using the “obvious” method from the formula, and using a Taylor series approximation.

1. Write down the Taylor series expansion for $f(x) = \cos x - 1$ around $x = 0$.
2. Write a matlab routine `accuratef` with the following specifications. `accuratef` takes a single real number x as the argument, and returns an accurate value of $f(x)$, evaluated using the Taylor series expansion. Your code should include a loop in which each iteration adds the next nonzero term in the Taylor series expansion to the current approximation of $f(x)$. The loop should terminate when the absolute value of the next nonzero term is less than 10^{-14} times the current approximate value of f .
3. Test your routine `accuratef` by writing a code `signif.m`, which calls `accuratef` 30 times with different values of x each time. Specifically, start with $x = 0.4$, and halve the value of x each time `accuratef` is called. (That is, call `accuratef` with $x = 0.4, 0.2, 0.1, 0.05, \dots$) For each value of x you should print out a single line that lists:
 - the value of x (to 5 digits of accuracy);
 - the value obtained from the “obvious” formula $f(x) = \cos x - 1$ (to 14 digits of accuracy);
 - the value obtained from `accuratef` (to 14 digits of accuracy);
 - the relative error in the “obvious” result (to 3 digits).
4. By making a simple change to the formula $\cos x - 1$ based on trig identities, you can obtain an alternative, equivalent formula for $f(x)$ that can be evaluated with little or no loss of significance for x near zero. Derive this form and explain why it is better.

Hand in hard copies of your codes and output together with your written answers to question 1 and question 4. Also, hand in your codes electronically as follows: Name your codes `accuratef.m` and `signif.m` (no other names accepted) and put them in a directory called `homework1`. From the parent directory of `homework1`, run the following command:

```
handin -c cs416-1 -a hwk1 -d homework1
```