

CS416 Spring 2007

Prof. Wright

Assignment #9

Due April 27, 2007

1. Consider the following linear program:

$$\begin{aligned} & \min x_1 + 3x_2 - x_3 \\ & \text{subject to } 4x_1 + 2x_2 + x_3 \geq 10, \\ & \quad 0 \leq x_3 \leq 20, \\ & \quad 2x_1 + x_2 + 4x_3 \leq 30, \\ & \quad x_1 \geq 0, \quad x_2 \geq 0. \end{aligned}$$

- (a) Reformulate as a linear program in standard form.
(b) Reformulate as a linear program in canonical form.
2. Consider the following overdetermined system of linear equations:

$$x_1 + 3x_2 = 5, \quad -2x_1 + x_2 = 3, \quad -x_1 - x_2 = -2.$$

- (a) Find the least-squares solution of this system by writing down and solving the normal equations.
(b) Write down the linear program whose solution yields the value of (x_1, x_2) that minimizes the ℓ_1 -norm of the violation of these equations.
(c) Write down the linear program whose solution yields the value of (x_1, x_2) that minimizes the ℓ_∞ -norm of the violation of these equations.

For (b) and (c) you can write the linear programs in “general” form, that is, no need to convert to either canonical or standard form.

3. The Madison Ale Company has been asked to supply drinks for a party of engineering graduate students. The company manager, Mr. Griewank, checks on availability of supplies and finds that his supplier of malt is having labor trouble and cannot make a delivery on time for the party. However, he finds that he had 75 units of malt stored in his factory, which he had bought earlier at a cost of \$0.80/unit. The suppliers of his other two ingredients, hops and yeast, are able to sell him whatever he needs at a cost of \$0.50/unit for hops and \$0.50/unit for yeast.

Griewank knows that his two products, Monona Dark and Mendota Light, require the following quantities of hops, yeast, and malt in order to make one gallon of each:

	Malt	Hops	Yeast
Mendota Light	2	3	2
Monona Dark	3	1	1.75

The beer sells at a price of \$8.00/gallon for the Light and \$10.00/gallon for the Dark. The students have asked for a maximum of 10 gallons of Monona Dark, but they can consume unlimited quantities of the Light.

Formulate the linear program that Griewank must solve in order to maximize his profits.