(i) If a problem has no solution, many solutions, or an unbounded objective function, you must clearly state so and justify your claim.

(ii) Solve each problem using as few pivots as possible. The whole exam can be solved by a total of 4 pivots only.

Last Name (Print):__________
First Name:__________

Grades

1. Question 1:____

2. Question 2:____

3. Question 3:____

4. Question 4:____

5. Total:____
1. Solve:

\[
\begin{align*}
  x_1 + x_3 + x_4 &= 1 \\
  x_1 + x_2 - x_3 + x_4 &= 1 \\
  -x_1 - 2x_2 + 3x_3 - x_4 &= 1
\end{align*}
\]
2. 

minimize \[-4x_1 + x_2 - 2x_3 - x_4\]
subject to 
\[-x_1 - x_2 + x_3 + x_4 \geq -2\]
\[-2x_1 - x_2 - x_3 + x_4 \geq -1\]
\[3x_1 + x_2 + 2x_3 + x_4 \geq -3\]
\[x_1, x_2, x_3, x_4 \geq 0\]
3. Solve \textbf{without} changing the number of variables or constraints:

\text{minimize} \quad 2x_1 - x_2 - x_3 \\
\begin{align*}
x_1 + 3x_2 & \geq 2 \\
3x_1 + 2x_2 & \geq 2 \\
x_1, x_2 & \geq 0
\end{align*}

subject to \quad \begin{align*}
-x_1 + x_2 - x_3 & = -1 \\
x_1 + x_2 + x_3 & \geq 0 \\
x_3: \text{unrestricted}
\end{align*}
Scratch Sheet
4.

minimize \[ 2x_1 + x_2 + 3x_3 \]
subject to
\[ 2x_1 + x_2 + x_3 \geq -1 \]
\[ -x_1 + x_2 + x_3 \geq 1 \]
\[ 3x_1 + x_2 + 2x_3 \geq -2 \]
\[ x_1, x_2, x_3 \geq 0 \]