Similar arguments to those above can be used to ascertain whether changes to the values of A_{ij} affect the optimal value and solution of the original problem. The issues are more complicated, however, since all parts of the tableau can be affected (including both the last column and last row), and we will not discuss them here.

Two other changes to the problem that might affect the optimal value of the linear program or the optimal solution vector are addition of a new variable and addition of a new constraint. In the next section, we use an analysis like that above to determine the effect of such changes to the linear program.

Exercise 6-1-2. Consider the following linear program

$$\begin{array}{llcl} \min & -x_1 - 4x_2 - x_3 & & \\ & 2x_1 + 2x_2 + x_3 & = & 4 \\ \text{subject to} & x_1 - x_3 & = & 1 \\ & x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0. & & \end{array}$$

1. Verify that an optimal basis for this problem is $\mathrm{B} = \{1, 2\}$, and calculate the quantities $\mathcal{A}_{\cdot\mathrm{B}}$, $\mathcal{A}_{\cdot\mathrm{B}}^{-1}$, $\mathcal{A}_{\cdot\mathrm{N}}$, p_{B}, p_{N}, and x_{B} for this basis, together with the reduced cost vector $c = p_{\mathrm{N}}' - p_{\mathrm{B}}'\mathcal{A}_{\cdot\mathrm{B}}^{-1}\mathcal{A}_{\cdot\mathrm{N}}$.

2. Suppose that the right-hand side 1 of the second constraint is replaced by $1+\epsilon$. Calculate the range of ϵ for which the basis B remains optimal, and give the solution x for each value of ϵ in this range.

3. Suppose that the coefficient of x_2 in the objective is replaced by $-4+\delta$. Find the range of δ for which the basis B remains optimal,

6.2 Adding New Variables or Constraints

Suppose we have solved a problem with an optimal basis B and we desire to add an extra variable with constraint matrix column $a \in \mathbf{R}^m$ and objective coefficient $\pi \in \mathbf{R}$, that is we now have

$$\begin{array}{ll} \min & p'x + \pi x_{l+1} \\ \text{subject to} & \mathcal{A}x + ax_{l+1} = b, \quad x, x_{l+1} \geq 0. \end{array}$$

To check whether adding this column affects the basis, we just calculate the corresponding reduced cost entry. If the entry is negative, that is,

$$\pi - p_{\mathrm{B}}'\mathcal{A}_{\cdot\mathrm{B}}^{-1}a < 0,$$

then the basis must be changed; otherwise it remains optimal.