CS726, Fall 2008
Homework 11
(for practice only; no need to submit)

1. Let H_k be a positive definite matrix and define $\bar{x}_k$ as

$$\bar{x}_k = \arg\min_{x \in \Omega} \nabla f(x_k)^T (x - x_k) + \frac{1}{2\alpha_k}(x - x_k)^T H_k (x - x_k).$$

Show that $\bar{x}_k$ solves the problem

$$\min_{x \in \Omega} \frac{1}{2} \left\| x - \left(x_k - \alpha_k H_k^{-1} \nabla f(x_k) \right) \right\|_{H_k}^2,$$

where the weighted norm $\| \cdot \|_{H_k}$ is defined by $\|z\|_{H_k} = \sqrt{z^T H_k z}$.

2. Use Theorem 2.3 in the notes to show that for $\bar{x}_k$ defined in Q1, we have that

$$\nabla f(x_k)^T (\bar{x}_k - x_k) \leq 0,$$

that is, $\bar{x}_k - x_k$ is a descent direction for f. Show moreover that this inequality is *strict* unless $\bar{x}_k = x_k$.

3. Consider the problem

$$\min f(x) \ \text{ subject to } l \leq x \leq u,$$

where l and u are finite vectors of lower and upper bounds, respectively. Find the first-order necessary conditions for optimality of this problem (extending the conditions (2.3) in the notes for the problem with constraints $x \geq 0$).