

CS / ISyE 730 - Spring 2011 - Homework 9

(assigned 4/21/11, due 4/29/11)

1. Suppose you are given have a semidefinite program in the following form:

$$\min_{y \in \mathbb{R}^m} b^T y \quad \text{s.t.} \quad \sum_{i=1}^m y_i A_i \succeq C, \quad y_i \geq 0 \quad (i = 1, 2, \dots, m), \quad \sum_{i=1}^m y_i = 1.$$

Find symmetric matrices $\tilde{A}_i, i = 1, 2, \dots, m$ and \tilde{C} such that ALL the constraints in this problem can be expressed in the following aggregated form:

$$\sum_{i=1}^m y_i \tilde{A}_i \succeq \tilde{C}.$$

2. Consider the SDP defined by the following data:

$$C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \quad b_1 = 0, \quad b_2 = 2.$$

(This was Example 2 studied in class, for which both primal and dual attain their optima but a duality gap remains.)

- Show that if we set $b_1 = \epsilon > 0$, then both primal and dual attain the same optimal value 0.
- Show that if we set $c_{22} = \epsilon > 0$, then both primal and dual attain the same optimal value 1.
- Show that if we make *both* these changes, then both primal and dual attain the same optimal value .75. (Hint: Solve the dual first, then try setting $x_{12} = .5$ in the primal and use weak duality.)

3. In the experiment design problem mentioned in the class notes, we formulate an SDP to maximize the minimum eigenvalue of the design matrix $A^T A$ by choosing the frequency λ_i with which to apply each probing vector a_i . (We have $A^T A = N \sum_{i=1}^m \lambda_i a_i a_i^T$, where m is the number of probing vectors and N is the total number of experiments to be performed.) Our SDP formulation for the problem of maximizing the minimum eigenvalue is:

$$\max_{t, \lambda} t \quad \text{s.t.} \quad \sum_{i=1}^m \lambda_i a_i a_i^T - tI \geq 0, \quad \sum_{i=1}^m \lambda_i = 1, \quad \lambda_i \geq 0, \quad i = 1, 2, \dots, M.$$

Write the SDP to solve a different problem: minimizing the trace of $(A^T A)^{-1}$.