New Directions 2016: Mathematical Optimization
August 2016
Worksheet 3

1. Prove Gale's theorem: Given $A \in \mathbb{R}^{p\times n}$ and $c \in \mathbb{R}^p$, we have either

$$(I) \qquad Ax \le c \qquad \text{for some } x \in \mathbb{R}^n$$

or

$$(II) \qquad A^T y = 0, \;\; c^T y = -1, \;\; y \ge 0 \qquad \text{for some } y \in \mathbb{R}^p,$$

but never both.

2. Write code in Matlab or Python (or a lower-level language) to solve linear programs using Algorithms LPF from Chapter 5 of *Primal-Dual Interior-Point Methods.* Problems are generated according to the codes `main.m` and `tester.m`, available from the web site. (If you are using a language other than Matlab, you'll have to translate these codes yourself.) Run your experiments on dense problems of size 50×200 that are generated by `tester.m`.

   - Experiment with different values of γ, $\sigma_{\min}$ and $\sigma_{\max}$.
   - Experiment with different heuristics for choosing σ_k at each iteration.
   - As well as choosing the steplengths rigorously to stay in the neighborhood $\mathcal{N}_{-\infty}(\gamma)$, try alternative heuristics that step .99 or .999 of the maximum α that can be taken before violating nonnegativity of x and s.

   The trickiest part of your code will be to solve the quadratics to find the maximum value of steplength α at each iteration that maintains membership of the neighborhood $\mathcal{N}_{-\infty}(\gamma)$. These quadratics have the following form:

$$(x_i + \alpha\Delta x_i)(s_i + \alpha\Delta s_i) \ge \gamma(x + \alpha\Delta x)^T(s + \alpha\Delta s)/n, \quad i = 1, 2, \ldots, n.$$

   You need to find the maximal $\alpha \in [0, 1]$ such that these inequalities are all satisfied.

   Describe your experiments, commenting in particular on the heuristics that you found to be most effective.