

1. Stirling's Formula: $n! \approx \sqrt{2\pi n} \left(\frac{n}{e}\right)^n$ large n .

2. $e = \lim_{h \rightarrow 0^+} (1+h)^{\frac{1}{h}}$.

3. l'Hospital's Rule: If $\lim_{x \rightarrow a} f(x) = 0$ and $\lim_{x \rightarrow a} g(x) = 0$ then

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

4. Maclaurin Series:

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \cdots + \frac{f^{(n)}(0)}{n!}x^n + \cdots.$$

Thus,

$$(a) \quad \frac{1}{1-p} = 1 + p + p^2 + \cdots + p^n + \cdots = \sum_{i=0}^{\infty} p^i \quad |p| < 1.$$

$$(b) \quad e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \cdots = \sum_{i=0}^{\infty} \frac{x^i}{i!} \quad |x| < \infty.$$

$$5. \quad \sum_{i=0}^k p^i = \frac{1-p^{k+1}}{1-p} \quad [\text{prove by induction on } k].$$

$$6. \quad \sum_{i=1}^n i = \frac{n(n+1)}{2} \quad [\text{prove by induction on } n]$$

$$7. \quad \text{Binomial Theorem: } (x+y)^n = \sum_{k=0}^n \binom{n}{k} x^k y^{n-k}.$$

$$[\text{Proof: (a) show } \binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}].$$

$$(b) \text{ prove } (x+y)^n \text{ by induction on } n.]$$