Helmholtz Stereopsis

A Surface Reconstruction Method

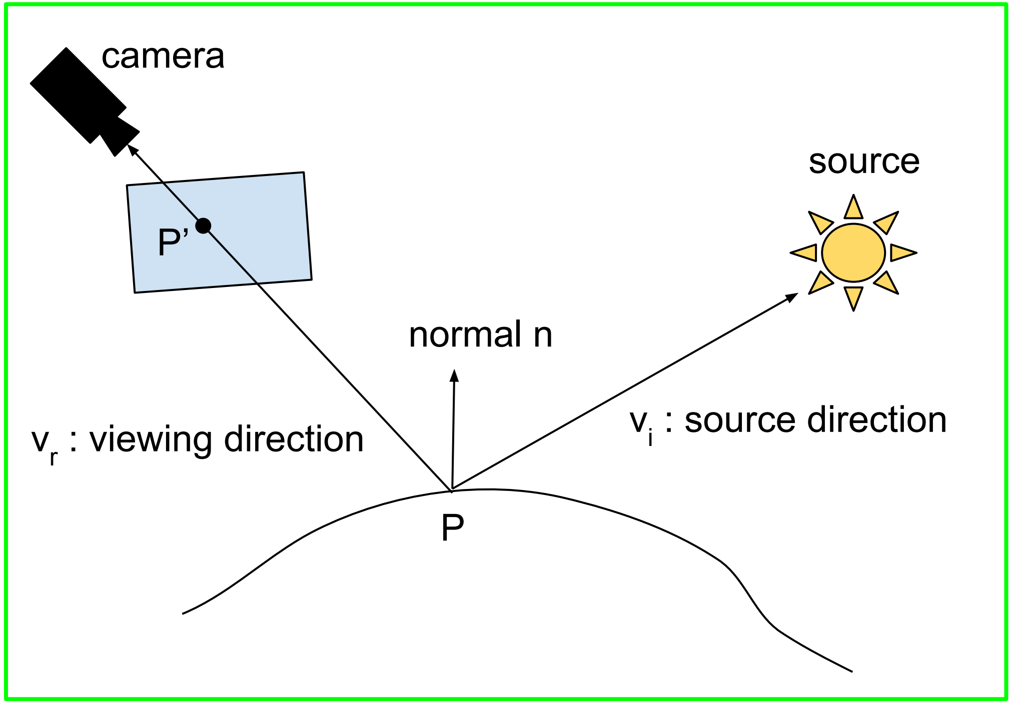
What is Helmholtz Stereopsis?

- A method for **3D surface reconstruction** (depth and normals)
- Other methods for surface reconstruction have some drawbacks
  - Stereo - Needs some kind of texture to be present in the scene
  - Photometric Stereo - Assumes a lambertian reflectance model
- Helmholtz Stereopsis makes **no assumption** about the reflectance properties of the surface

Review

- <u>Surface Irradiance L</u>
  A measure of intensity **received** by point P from the source

- <u>Surface Radiance I</u>
  A measure of intensity **emitted** by the point P towards the camera



The surface radiance at P due to a point source with unit intensity located at position $\mathbf{O}_i$, can be calculated as follows:

$$Surface\ Irradiance\ L(v_i) = \frac{\hat{n} \cdot \hat{v}_i}{|O_i - P|^2}$$

Review

- BRDF (bidirectional reflectance distribution function)
  - Material property
  - Function of the lighting and viewing directions
  - Ratio of Irradiance $I(v_r)$ and Radiance $L(v_i)$

$$BRDF(v_i, v_r) = \frac{Radiance\ I(v_r)}{Irradiance\ L(v_i)}$$

- From these equations, we can write the following:

$$I(v_r) = BRDF(v_i, v_r)\frac{\hat{n}\cdot\hat{v}_i}{|O_i - P|^2}$$

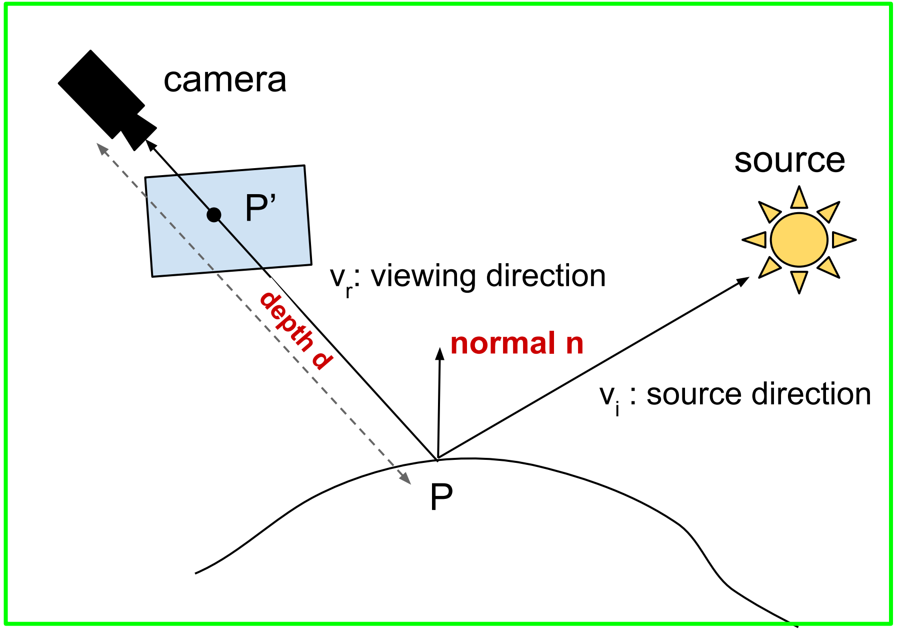
- Lambertian surfaces (***constant*** BRDF) emit equal amount of light in all directions

Helmholtz Reciprocity

$$BRDF(v_i, v_r) = BRDF(v_r, v_i)$$

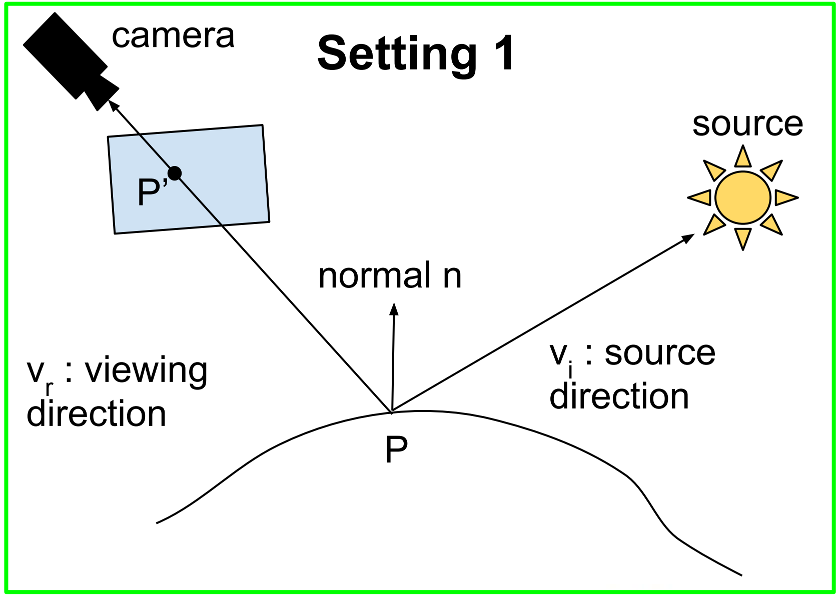
Interchanging the **lighting** and the **viewing** directions does not change the BRDF value

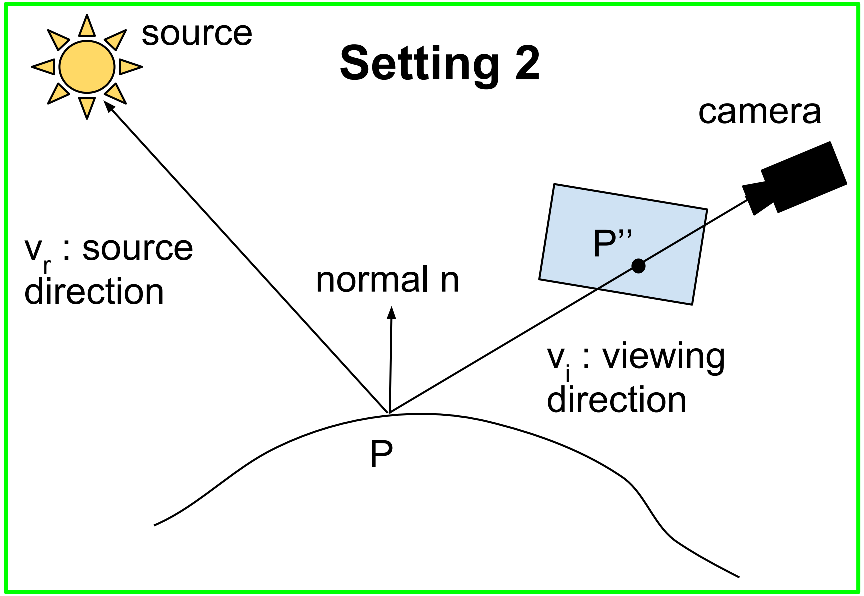
Revisiting the problem



Given the camera position, source position and pixel intensity at **pixel P'**, we want to determine the **depth** of the corresponding 3D point P and **surface normal n**

Reciprocal pair





$$I_r = BRDF(v_i, v_r)\frac{\hat{n}\cdot\hat{v}_i}{|O_i - P|^2}$$

$$I_i = BRDF(v_r, v_i)\frac{\hat{n}\cdot\hat{v}_r}{|O_r - P|^2}$$

Eliminating the BRDF term, we get

$$\left(I_r\frac{\hat{v}_r}{|O_r - P|^2} - I_i\frac{\hat{v}_i}{|O_i - P|^2}\right)\cdot\hat{n} = 0$$

Exploiting the constraint

$$\left(I_r \frac{\hat{v}_r}{|O_r - P|^2} - I_i \frac{\hat{v}_i}{|O_i - P|^2}\right) \cdot \hat{n} = 0$$

- We know $\mathbf{O}_r$ and $\mathbf{O}_i$ (camera/source positions)
- Given a pixel **P'**, we know $\mathbf{I}_r$
- The values $\mathbf{v}_r$, $\mathbf{v}_i$, **P** and $\mathbf{I}_i$ depend only on depth **d (unknown)**
- Surface normal **n** (**unknown)**

$$w(d) \cdot \hat{n} = 0$$

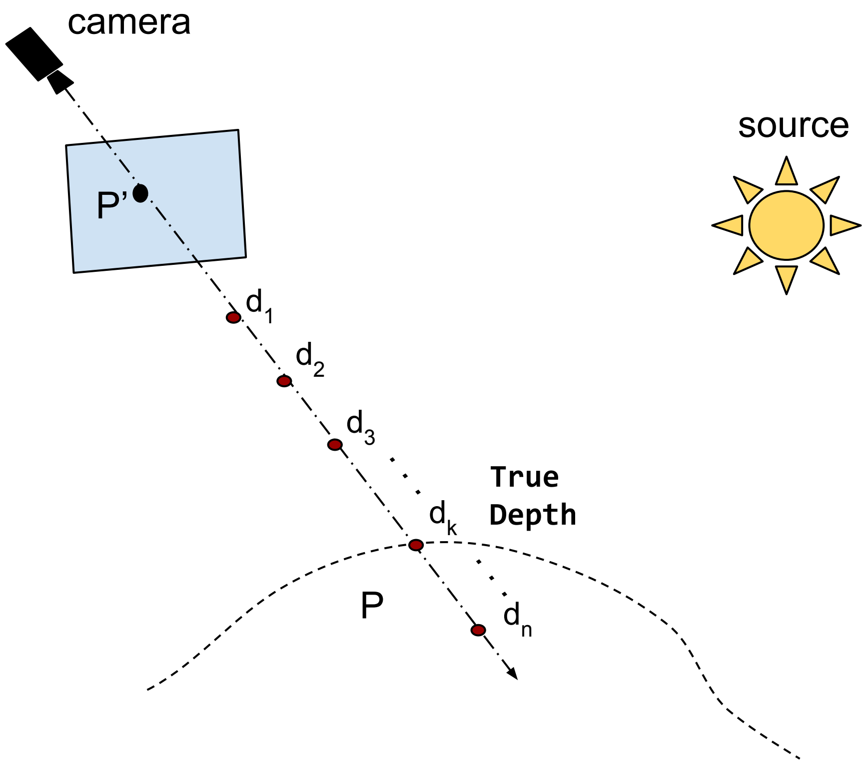
Exploiting the constraint

- We can use **3 reciprocal pairs** to get 3 different equations

$$
\begin{aligned}
w_1(d) \cdot \hat{n} &= 0 \\
w_2(d) \cdot \hat{n} &= 0 \\
w_3(d) \cdot \hat{n} &= 0
\end{aligned}
\quad\longrightarrow\quad
\underbrace{\begin{pmatrix} w_1(d) \\ w_2(d) \\ w_3(d) \end{pmatrix}_{3\times3}}_{W}
\begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix}_{3\times1}
=
\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}
$$

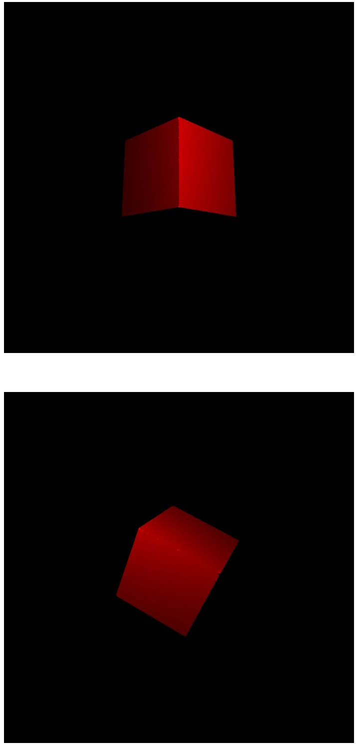
- For the true depth (**d***), the above system of equations will be satisfied
- Surface normal lies in the null space of **W**
- Implying, matrix **W** should be rank-2 for the correct value of **d**

Probing over depth

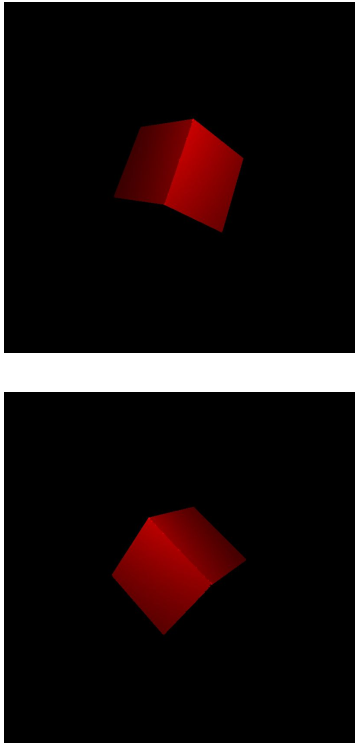


- Search over a set of d values d_1, d_2, d_3, ... d_n
- Construct the W matrix for each d_i and look at its rank
- The d_i that results in a rank-2 matrix is "the one"
- Repeat this process for every pixel to get the entire depth map

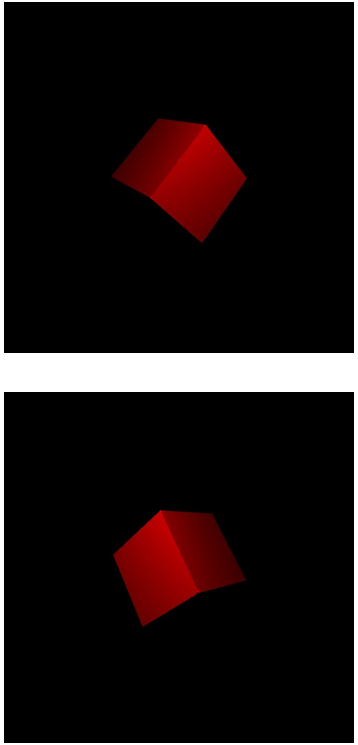
Results



Reciprocal Pair 1

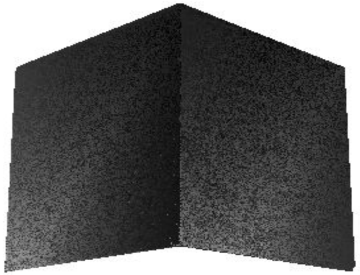


Reciprocal Pair 2

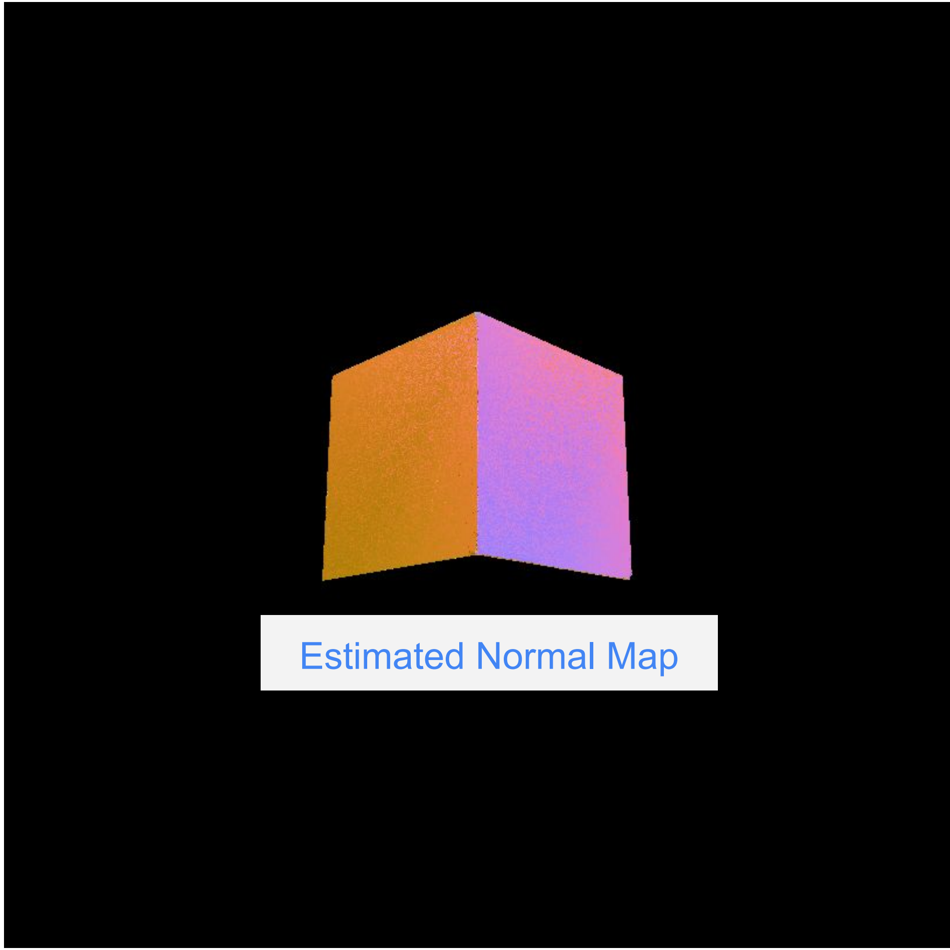


Reciprocal Pair 3

Results



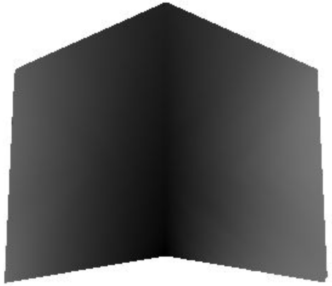
Estimated Depth Map



Estimated Normal Map

Lambertian cube 3 pairs

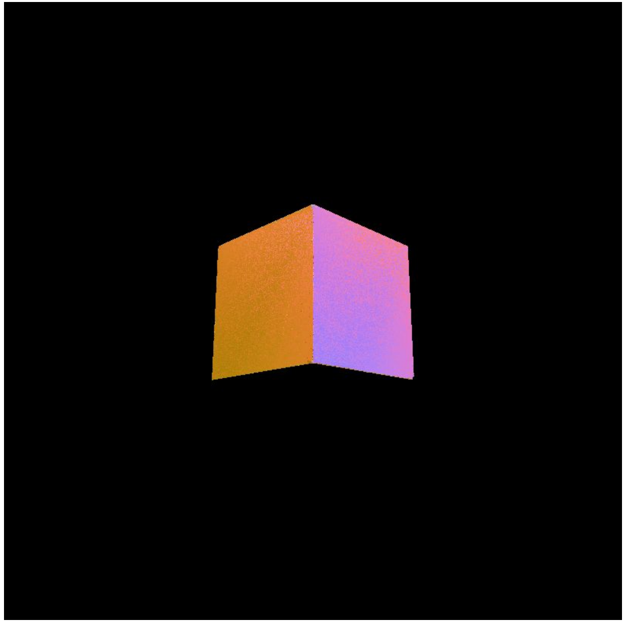
Results



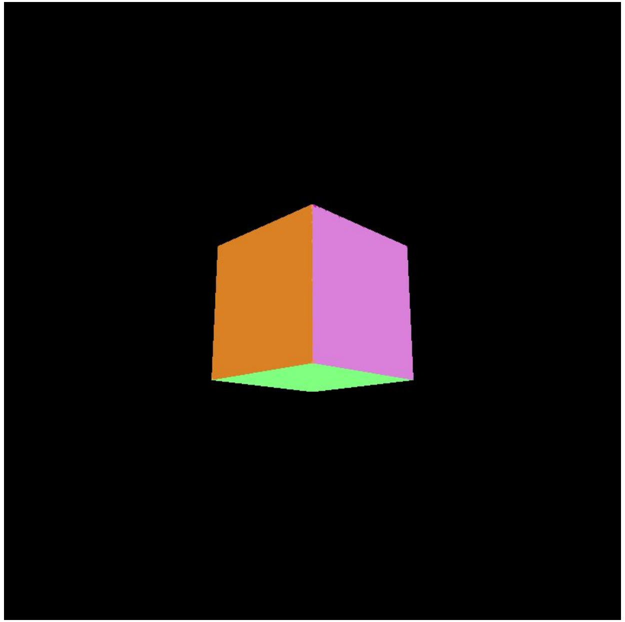
Depth from Normals

Lambertian cube 3 pairs

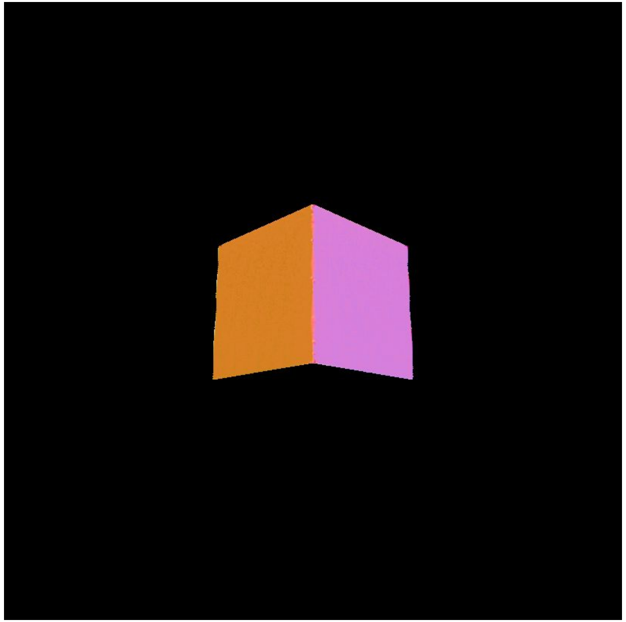
Results



Using 3 pairs

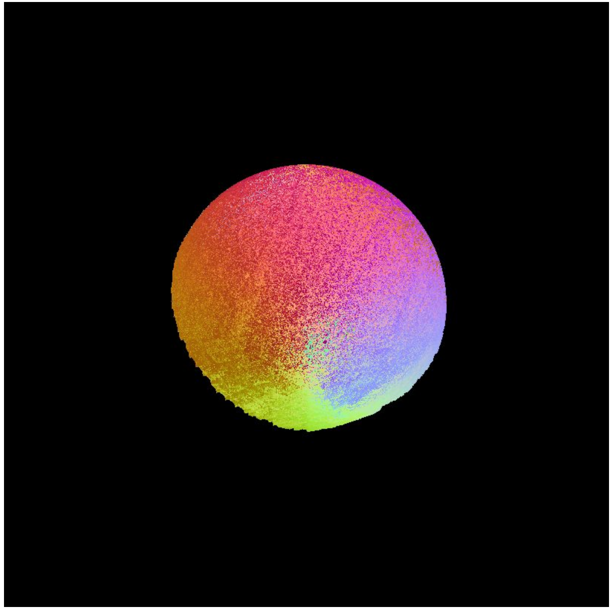


True Normal Map

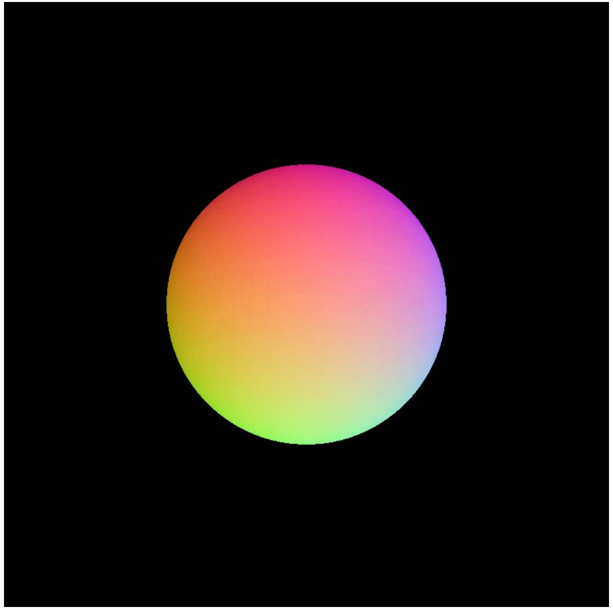


Using 20 pairs

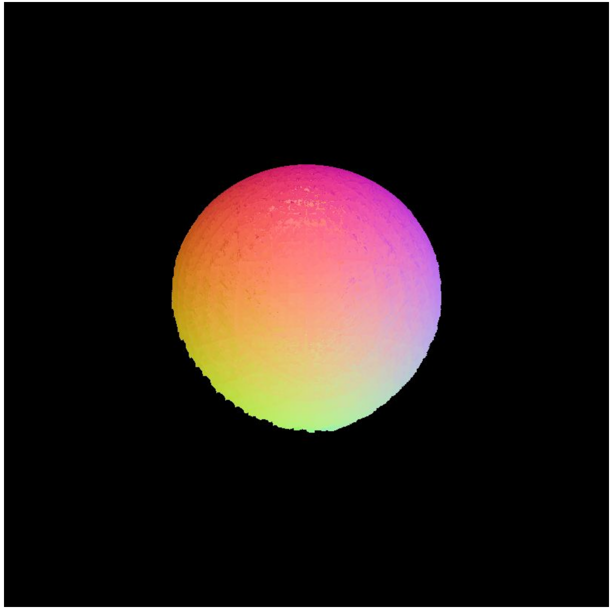
Results



Using 3 pairs

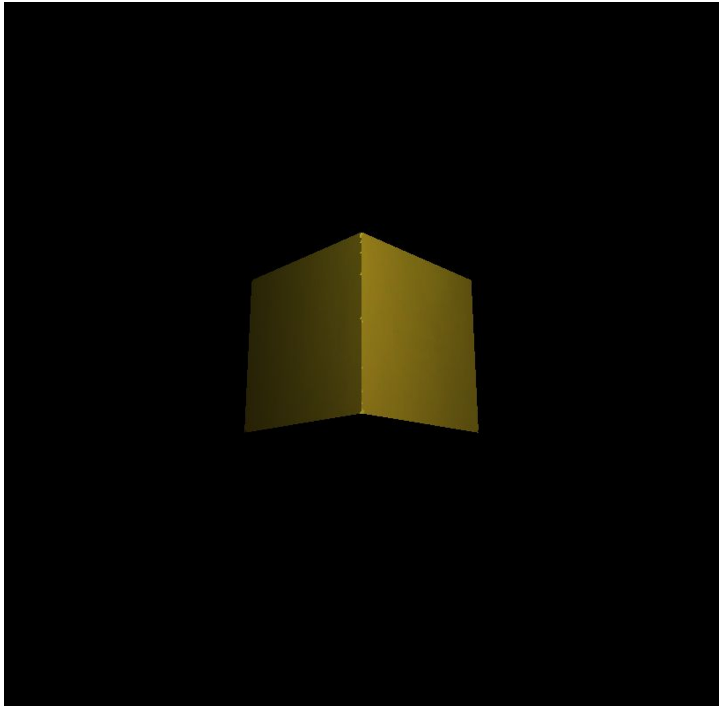


True Normal Map

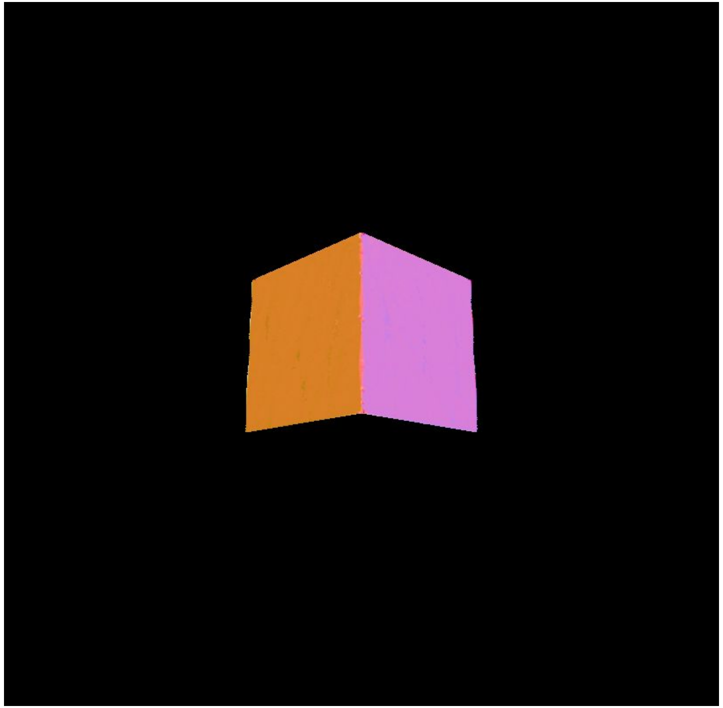


Using 20 pairs

Results



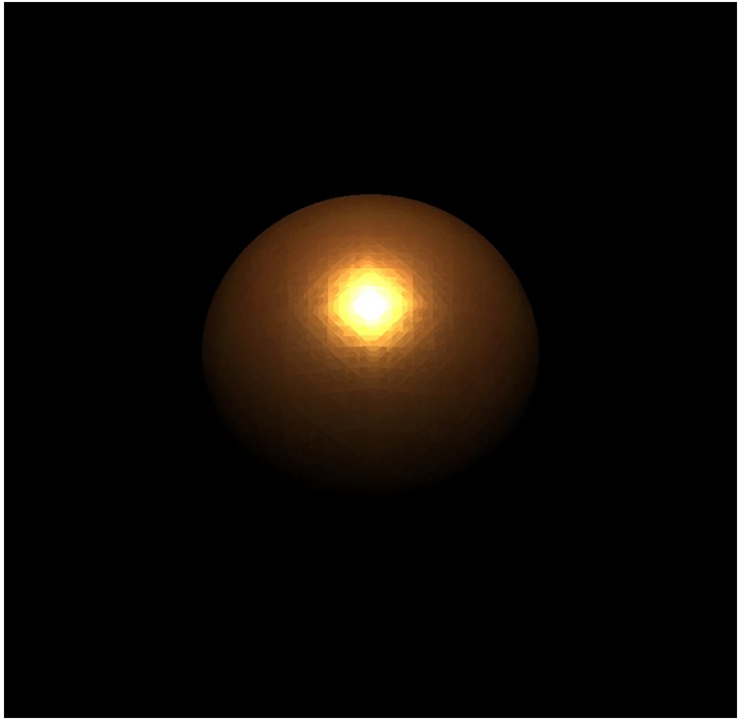
Principal Camera



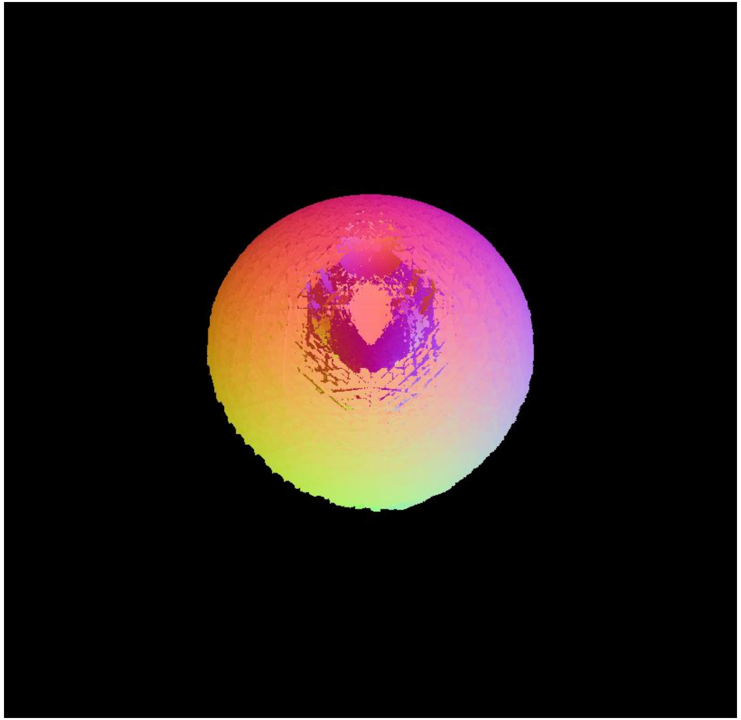
Estimated Normal Map

Plastic cube 20 pairs

Results



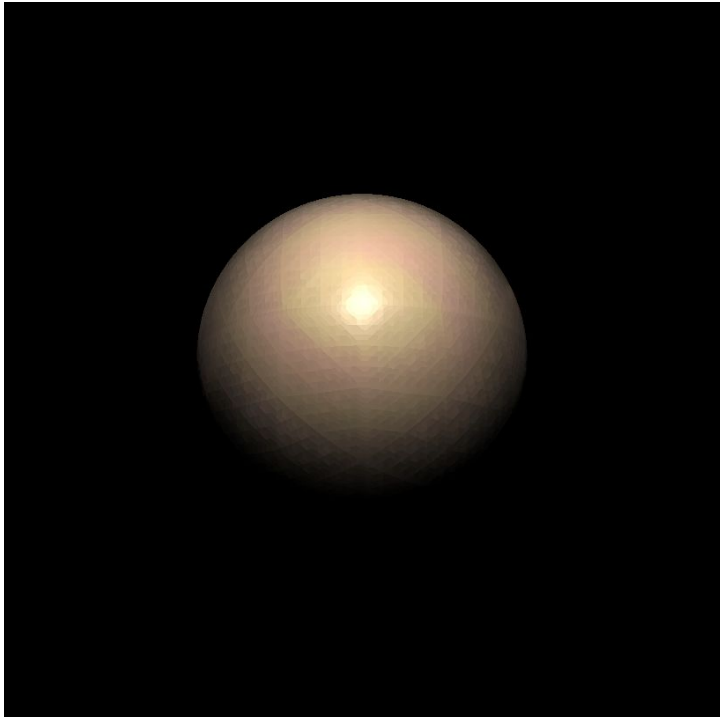
Principal Camera



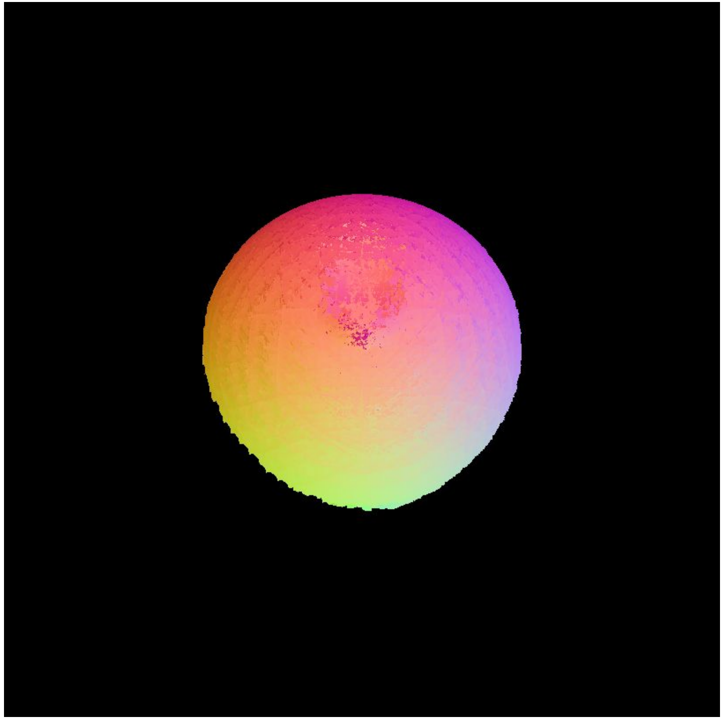
Estimated Normal Map

Gold sphere 20 pairs

Results



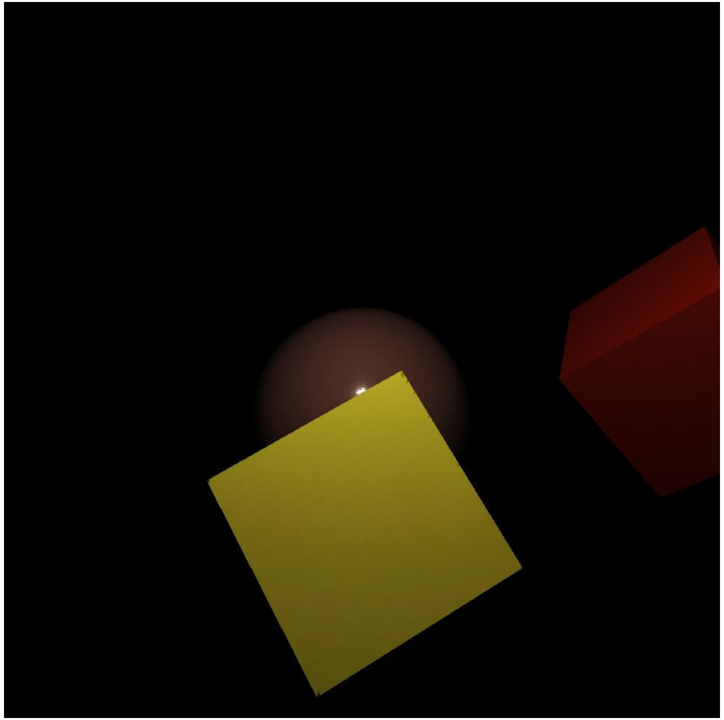
Principal Camera

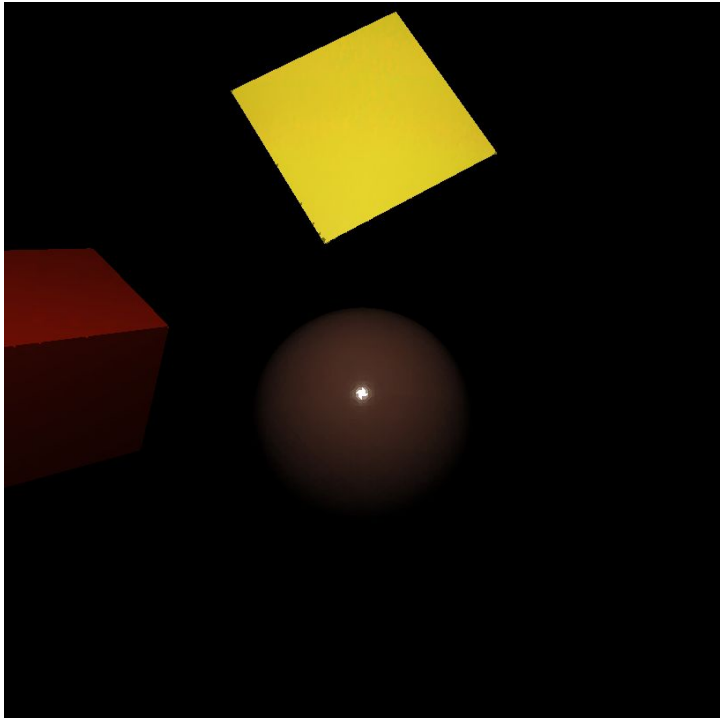


Estimated Normal Map

Rubber sphere 20 pairs

Results

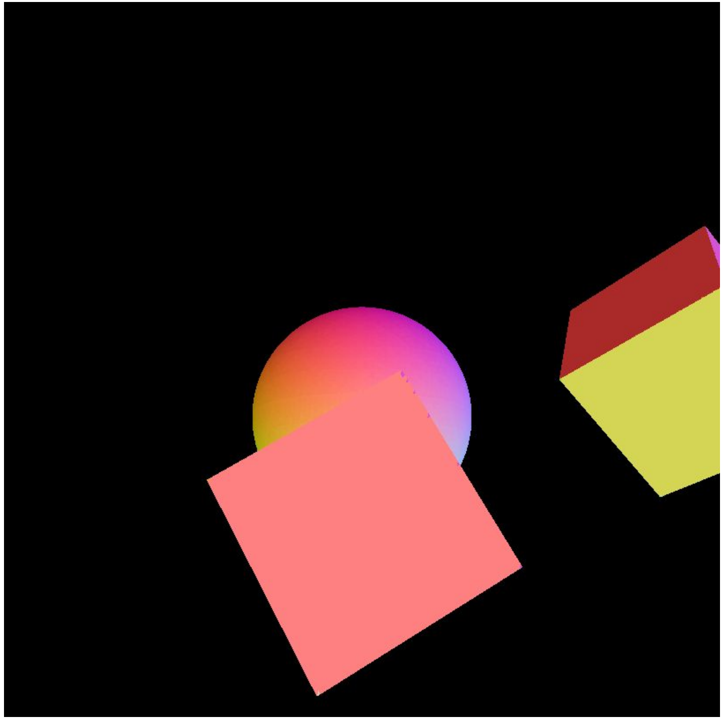




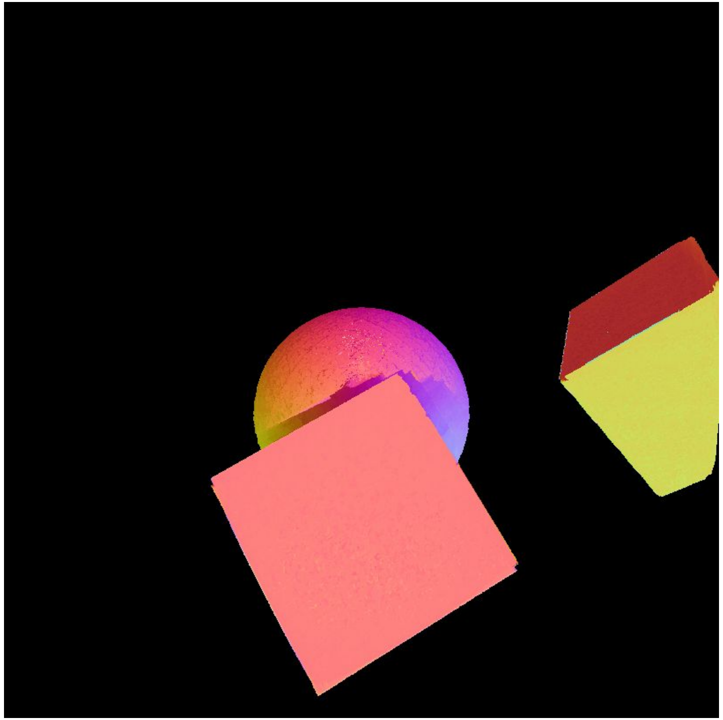
Reciprocal Pair

Compound scene 20 pairs

Results



True Normal Map



Estimated Normal Map

Compound scene 20 pairs

References

[1] T. Zickler, P.N. Belhumeur, and D.J. Kriegman. Helmholtz Stereopsis: Exploiting Reciprocity for Surface Reconstruction. In Proc. of the ECCV, page III: 869 ff., 2002

[2] https://www.merl.com/brdf/

[3] Frankot, R.T., Chellappa, R.: A method for enforcing integrability in shape from shading algorithms. IEEE Trans. Pattern Anal. Machine Intell. 10 (1988) 439–451

Singular value decomposition

- In practice, it is not possible to get a **W** matrix that is exactly rank-2
- We compare the ratio of sigma2/sigma3. Higher the ratio, closer the matrix is to being rank-2
- We select that **d** value which corresponds to the highest sigma2/sigma3 ratio
- Once we know **d***, the **normal** can be recovered by taking the rightmost singular vector of the corresponding W matrix