

# Validating Library Usage Interactively

**William Harris**, Guoliang Jin, Shan Lu, and Somesh Jha



# Outline

**1. Motivation**

2. Problem definition

3. Technique and experiments

# Outline

1. Motivation
2. Problem definition
3. Technique and experiments

# Outline

1. Motivation

3. Technique and experiments

# Outline

## I. Motivation

# Developers Often Optimize **Library** Usage

# Developers Often Optimize **Library** Usage

In a previous study (*Understanding and Detecting Real-World Performance Bugs*, Jin et. al., PLDI '12),  
of 109 bug patches:

# Developers Often Optimize **Library** Usage

In a previous study (*Understanding and Detecting Real-World Performance Bugs*, Jin et. al., PLDI '12),  
of 109 bug patches:

**42**

caused by

"an inefficient function call  
sequence composed of efficient individual functions"



# Developers Often Optimize **Library** Usage

In a previous study (*Understanding and Detecting Real-World Performance Bugs*, Jin et. al., PLDI '12),  
of 109 bug patches:

**34**

caused by

"calling functions that conduct  
unnecessary work given the calling context"

# Developers Often Optimize **Library** Usage *Concisely*

In a previous study (*Understanding and Detecting Real-World Performance Bugs*, Jin et. al., PLDI '12),  
of 109 bug patches:

**42**

contained  $\leq 5$  lines of code,  
and median size was 8 lines

# The Case for Verification

Opportunities to improve practice: optimizations...

- are discussed extensively
- validated by testing only

Why its feasible: optimizations...

- use a clean interface
- are small

# The Case for Verification

Opportunities to improve practice: optimizations...

- are discussed extensively
- validated by testing only

Why its feasible: optimizations...

- use a clean interface
- are small

Why its hard: optimizations implicitly rely on subtle library properties

# Mozilla Bug #40996 I

## Original

```
DbList dbnewList;  
Card card;  
for (pos=1;pos<=numAddrs;pos++)  
    card = CreateCard(user(pos));  
    dbnewList = DbList();  
    AddAddrToList(dbnewList,card);  
return card;
```

## Optimized

# Mozilla Bug #40996 I

## Original

```
DbList dbnewList;  
Card card;  
for (pos=1;pos<=numAddrs;pos++)  
    card = CreateCard(user(pos));  
    dbnewList = DbList();  
    AddAddrToList(dbnewList,card);  
return card;
```

## Optimized

# Mozilla Bug #40996 I

$\forall x, y, z.$

`AddAddressToList[arg2](x, y)`  
`= AddAddressToList[arg2](z, y)`

## Original

## Optimized

```
DbList dbnewList;
Card card;
for (pos=1; pos<=numAddrs; pos++)
    card = CreateCard(user(pos));
dbnewList = DbList();
AddAddrToLst(dbnewList, card);
return card;
```

# MySQL Bug #38769

## Original

```
for (n=0; n < 256; ++n) {  
    Snapshot_info* i = at(info, n);  
    if (!i) continue;  
    process_snapshot(i);  
}
```

## Optimized



# MySQL Bug #38769

## Original

```
for (n=0; n<< 256; ++n) { ;++n){  
    Snapshot_info* i = at(info, n);  
    if (!i) continue;  
    process_snapshot(i);  
}
```

## Optimized

# MySQL Bug #38769

## Original

```
for (n=0; n<< 256; ++n) { ;++n){  
    Snapshot_info* i = at(info, n);  
    if (!i) continue;  
    process_snapshot(i);  
}
```

## Optimized

```
    snap_count(info)
```

# MySQL Bug #38769

$\forall x. \text{snap\_count}(x) < 256$   
 $\forall x, y. \text{snap\_count}(x) \leq y \Rightarrow \text{at}(x, y) = \text{NULL}$

## Original

```
for (n=0; n<< 256; ++n) {  
    Snapshot_info* i = at(info, n);  
    if (!i) continue;  
    process_snapshot(i);  
}
```

## Optimized

```
    snap_count(info)
```

# Apache Bug #34464

## Original

```
string sb := "";  
while (!is_sub(sb, s)) {  
    app(sb, get());  
}  
return sb;
```

## Optimized

# Apache Bug #34464

## Original

```
string sb := "";  
while (!is_sub(sb, s)) {  
    app(sb, get());  
}  
return sb;
```

## Optimized

```
int pos := -len(s);  
    pos < 0  
    ||      sub( pos),  
pos := len(sb) - len(s);
```

# Apache Bug #34464

`len("") = 0`

$\forall x, y. \text{len}(x) > \text{len}(y) \Rightarrow \neg \text{is\_sub}(x, y)$

$\forall x, y, z. \neg \text{is\_sub}(x, y) \Rightarrow$

$\text{is\_sub}(\text{app}(x, z), y) \Leftrightarrow$

$\text{is\_sub}(\text{sub}(\text{app}(x, z), \text{len}(\text{app}(x, z)) - \text{len}(y)), y)$

## Original

```
string sb := "";  
while (!is_sub(sb, s)) {  
    app(sb, get());  
}  
return sb;
```

## Optimized

```
int pos := -len(s);  
    pos < 0  
    ||      sub(    pos),  
pos := len(sb) - len(s);
```

# Outline

1. Motivation

2. Problem definition

1. Problem definition

3. Technique and experiments

# Outline

2. Problem definition
1. Problem definition
3. Technique and experiments



# Outline

1. Problem definition
3. Technique and experiments

# Outline

## I. Problem definition

# Apache Bug #34464

Org.

```
string sb := "";  
while (!is_sub(sb, s)) {  
    app(sb, get());  
}  
return sb;
```

Opt.

```
string sb := "";  
int pos = -len(s);  
while (pos < 0 ||  
        !is_sub(sub(sb, pos), s)) {  
    app(sb, get());  
    pos := len(sb) - len(s);  
}  
return sb;  
||
```

# Org.

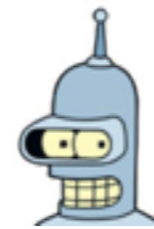
```
string sb := "";  
while (!is_sub(sb, s)) {  
    app(sb, get());  
}  
return sb;
```

# Opt.

```
string sb := "";  
int pos = -len(s);  
while (pos < 0 ||  
        !is_sub(sub(sb, pos), s)) {  
    app(sb, get());  
    pos := len(sb) - len(s);  
}  
return sb;  
||
```

↓ ↓

Org. **chklibs** Opt.



```
string sb := "";  
while (!is_sub(sb, s)) {  
    app(sb, get());  
}  
return sb;
```

```
string sb := "";  
int pos = -len(s);  
while (pos < 0 ||  
        !is_sub(sub(sb, pos), s)) {  
    app(sb, get());  
    pos := len(sb) - len(s);  
}  
return sb;  
||
```



chklibs 

Org.

Opt.

```

string sb := "";
while (!is_sub(sb, s)) {
  app(sb, get());
}
return sb;

```

```

string sb := "";
int pos = -len(s);
while (pos < 0 ||
      !is_sub(sub(sb, pos), s)) {
  app(sub(s, pos, pos + 1));
  pos = sub(sb) - len(s);
}
return sb;

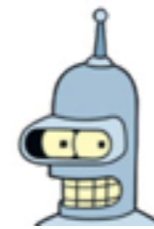
```

User



||

chklibs



Org.

Opt.

```

  ∀ x, y. is_sub(x, y)
string sb := "";
while (!is_sub(sb, s)) {
  app(sb, get());
}
return sb;

```

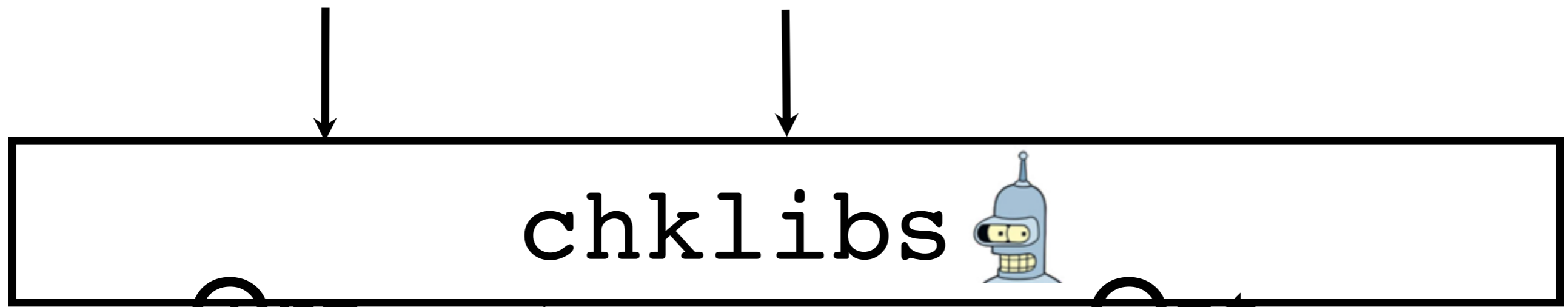
User



```

string sb := "";
int pos = -len(s);
while (pos < 0 ||
      !is_sub(sub(sb, pos), s)) {
  app(sub(s, pos, len(s) - pos), sb);
  pos = sub(sb) - len(s);
}
return sb;

```



chklibs 

Org.

Opt.

```

 $\forall x, y. \text{is\_sub}(x, y)$ 
string sb := "";
while (!is_sub(sb, Invalid
  app(sb, get()));
}
return sb;

```

```

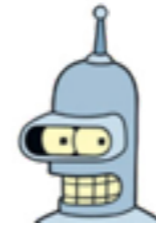
string sb := "";
int pos = -len(s);
while (pos < 0 ||
  !is_sub(sub(sb, pos), s)) {
  app(sb, get());
  pos = sub(sb) - len(s);
}
return sb;

```

User 



chklibs



Org.

Opt.

$\forall x, y. \text{len}(x) > \text{len}(y) \Rightarrow$

$\forall x, y. \text{is\_sub}(x, y)$

string sb := "";

while (!is\_sub(sb, Invalid

app(sb, get()));

}  
return sb;

Invalid

$\neg \text{is\_sub}(x, y)$  ";

int pos = -len(s);

while (pos < 0 ||

!is\_sub(sub(sb, pos), s)) {

app(sub(sb, pos), s);

pos = sub(sb, pos) - len(s);

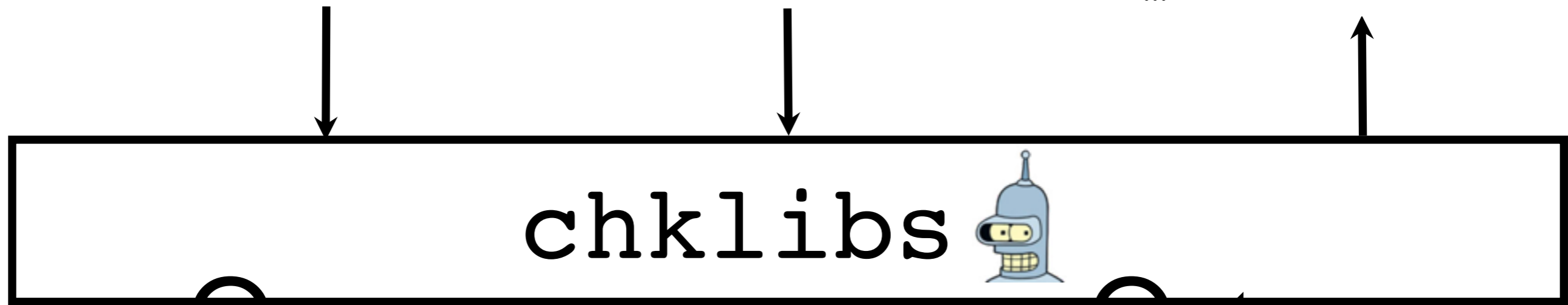
User



return sb,

||

Org.  $\approx$  Opt  
 assuming  
 $\forall x, y. \text{len}(x) > \text{len}(y) \Rightarrow$   
 $\neg \text{is\_sub}(x, y)$   
 ...



Org.

$\forall x, y. \text{len}(x) > \text{len}(y) \Rightarrow$

Opt.

$\forall x, y. \text{is\_sub}(x, y)$

$\neg \text{is\_sub}(x, y)$

string sb := "";

while (!is\_sub(sb, Invalid

app(sb, get());

}  
return sb;

Invalid

int pos = -len(

while (pos < 0

!is\_sub(sub(sb, pos), s)) {

app(sub(sb, pos), s);

pos = pos - len(s);

User



return sb;  
||

Valid

# chklibs Requirements

# chklibs Requirements

## I. Relative soundness

# chklibs Requirements

## I. Relative soundness

# chklibs

# Requirements

1. Relative soundness
1. Non-redundant queries
2. Queries on properties of the **library** only

# chklibs Requirements


## I. Non-redundant queries

# I. Non-redundant queries

## Inputs

```
string sb := "";  
while (!is_sub(sb, s)) {  
    app(sb, get());  
}  
return sb;
```

```
string sb := "";  
int pos = -len(s);  
while (pos < 0  
    !is_sub(sub(sb, pos), s)) {  
    app(sb, get());  
    pos := len(sb) - len(s);  
}  
return sb;
```

chklibs 

User 




# I. Non-redundant queries

## Inputs

```
string sb := "";  
while (!is_sub(sb, s)) {  
    app(sb, get());  
}  
return sb;
```

```
string sb := "";  
int pos = -len(s);  
while (pos < 0  
    !is_sub(sub(sb, pos), s)) {  
    app(sb, get());  
    pos := len(sb) - len(s);  
}  
return sb;
```

chklibs 

$\forall x, y. \text{len}(x) > \text{len}(y) \Rightarrow$   
 $\text{is\_sub}(x, y)$


User 

# I. Non-redundant queries

## Inputs

```
string sb := "";  
while (!is_sub(sb, s)) {  
    app(sb, get());  
}  
return sb;
```

```
string sb := "";  
int pos = -len(s);  
while (pos < 0  
    !is_sub(sub(sb, pos), s)) {  
    app(sb, get());  
    pos := len(sb) - len(s);  
}  
return sb;
```

chklibs 

$\forall x, y. \text{len}(x) > \text{len}(y) \Rightarrow$   
 $\text{is\_sub}(x, y)$

Invalid


User 

# I. Non-redundant queries

## Inputs

```
string sb := "";  
while (!is_sub(sb, s)) {  
  app(sb, get());  
}  
return sb;
```

```
string sb := "";  
int pos = -len(s);  
while (pos < 0  
  !is_sub(sub(sb, pos), s)) {  
  app(sb, get());  
  pos := len(sb) - len(s);  
}  
return sb;
```

chklibs 

$\forall x, y. \text{len}(x) > \text{len}(y) \Rightarrow$   
 $\text{is\_sub}(x, y)$

$\forall x, y. \text{is\_sub}(x, y)$

Invalid


User 

# I.Queries on properties of the **library** only

## Inputs

```
string sb := "";  
while (!is_sub(sb, s)) {  
    app(sb, get());  
}  
return sb;
```

```
string sb := "";  
int pos = -len(s);  
while (pos < 0  
    !is_sub(sub(sb, pos), s)) {  
    app(sb, get());  
    pos := len(sb) - len(s);  
}  
return sb;
```

chklibs 


User 

# I. Queries on properties of the **library** only

## Inputs

```
string sb := "";  
while (!is_sub(sb, s)) {  
  app(sb, get());  
}  
return sb;
```

```
string sb := "";  
int pos = -len(s);  
while (pos < 0  
  !is_sub(sub(sb, pos), s)) {  
  app(sb, get());  
  pos := len(sb) - len(s);  
}  
return sb;
```

chklibs 

$\forall x, y. \text{len}(x) > \text{len}(y) \Rightarrow$   
 $\neg \text{is\_sub}(x, y)$


User 

# I. Queries on properties of the **library** only

## Inputs

```
string sb := "";  
while (!is_sub(sb, s)) {  
  app(sb, get());  
}  
return sb;
```

```
string sb := "";  
int pos = -len(s);  
while (pos < 0  
  !is_sub(sub(sb, pos), s)) {  
  app(sb, get());  
  pos := len(sb) - len(s);  
}  
return sb;
```

chklibs 

$\forall x, y. \text{len}(x) > \text{len}(y) \Rightarrow$   
 $\neg \text{is\_sub}(x, y)$

$\text{pos} = \text{len}(\text{sb}) - \text{len}(s)$



User 

# Why involve the user?

- Libraries may be difficult to analyze
- Do small optimizations contain specifications of big callees?

# Outline

1. Motivation
2. Problem definition
- 3. Technique and experiments**



# Outline

2. Problem definition

B. **Technique** and experiments

# Outline

## B. **Technique** and experiments

# Outline

## I. Technique and experiments

# Technique

- Based on known translation-validation algorithm (Necula, PLDI '00)
- Key difference: simultaneously find formula in simulation relation and supporting **library** property over predicates in post

```
L0:  
string sb := "";  
L1:  
while (!is_sub(sb, s)) {  
    app(sb, get());  
}  
L2:  
return sb;
```

```
L0':  
string sb := "";  
int pos = -len(s);  
L1':  
while (pos < 0 ||  
        !is_sub(sub(sb, pos), s)) {  
    app(sb, get());  
    pos := len(sb) - len(s);  
}  
L2':  
return sb;
```



chklibs

User

# chklibs

```
L0:  
string sb := "";  
L1:  
while (!is_sub(sb, s)) {  
    app(sb, get());  
}  
L2:  
return sb;
```

```
L0':  
string sb := "";  
int pos = -len(s);  
L1':  
while (pos < 0 ||  
        !is_sub(sub(sb, pos), s)) {  
    app(sb, get());  
    pos := len(sb) - len(s);  
}  
L2':  
return sb;
```

validated [Lib](#).property:True

Uşer

# chklibs

```
L0: string sb := "";
L1: while (!is_sub(sb, s)) {
    app(sb, get());
}
L2: return sb;

L0': string sb := "";
    int pos = -len(s);
L1': while (pos < 0 ||
    !is_sub(sub(sb, pos), s)) {
    app(sb, get());
    pos := len(sb) - len(s);
}
L2': return sb;
```

validated Lib. property: True

U<sub>9</sub>ser

# chklibs

$s = s'$

```
L0: string sb := "";
L1: while (!is_sub(sb, s)) {
    app(sb, get());
}
L2: return sb;
```

$\rightarrow$  L0':  
string sb := "";  
int pos = -len(s);  
 $\rightarrow$  L1':  
while (pos < 0 ||  
!is\_sub(sub(sb, pos), s)) {  
 app(sb, get());  
 pos := len(sb) - len(s);  
}  
 $\rightarrow$  L2':  
return sb;

validated Lib. property: True

U<sub>9</sub>ser



# chklibs

```

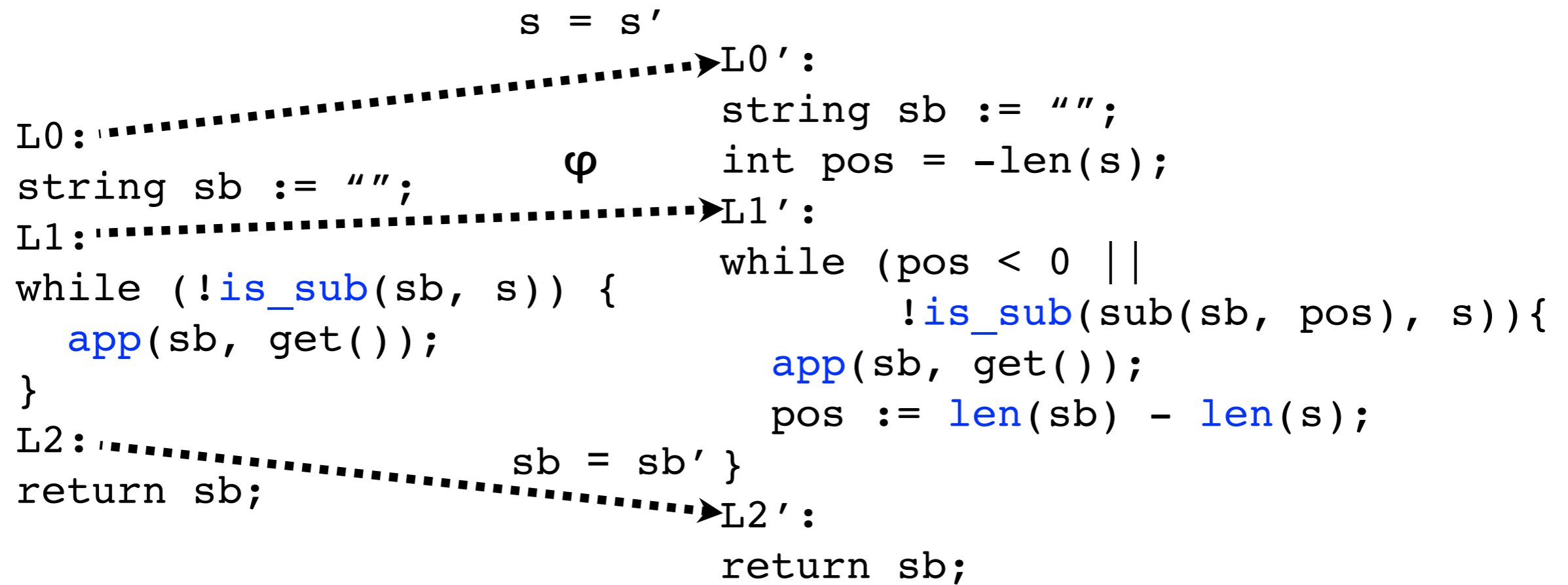
                                s = s'
L0: .....>L0':
string sb := "";                string sb := "";
                                int pos = -len(s);
L1: .....>L1':
while (!is_sub(sb, s)) {        while (pos < 0 ||
                                !is_sub(sub(sb, pos), s)) {
    app(sb, get());              app(sb, get());
}                                pos := len(sb) - len(s);
L2: .....>L2':
return sb;                       sb = sb' }
                                return sb;

```

validated Lib. property: True

Uşer

# chklibs



validated Lib. property: True

U<sub>9</sub>ser

# chklibs

```

                                s = s'
                                ↗
L0: string sb := "";           L0': string sb := "";
                                ↘
                                ϕ
L1: while (!is_sub(sb, s)) {    L1': while (pos < 0 ||
                                ↗
                                ↘
                                sb = sb' }
L2: return sb;                 L2': return sb;

```

$\text{post} \left[ \begin{array}{l} L1 \rightarrow L2 \\ L1' \rightarrow L2' \end{array} \right] ( \quad ) \Rightarrow sb = sb'$

validated `Lib`.property: True

Uşer

# chklibs

```

                                s = s'
                                L0':
                                string sb := "";
                                int pos = -len(s);
L0: string sb := "";
                                sb=sb'
                                L1':
                                while (pos < 0 ||
                                !is_sub(sub(sb, pos), s)) {
                                app(sb, get());
                                pos := len(sb) - len(s);
L1: while (!is_sub(sb, s)) {
                                app(sb, get());
                                }
                                sb = sb' }
L2: return sb;
                                L2':
                                return sb;

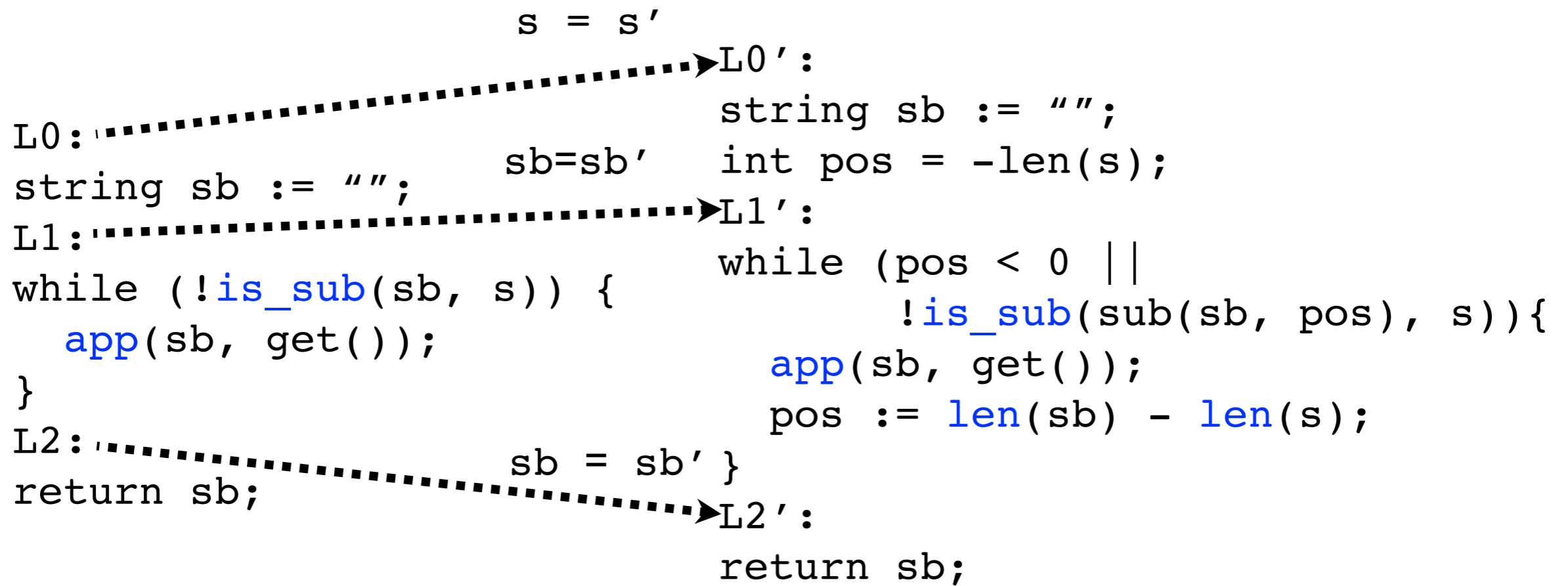
```

$\text{post} \left[ \begin{array}{l} L1 \rightarrow L2 \\ L1' \rightarrow L2' \end{array} \right] (sb = sb') \Rightarrow sb = sb'$

validated `Lib`.property: True

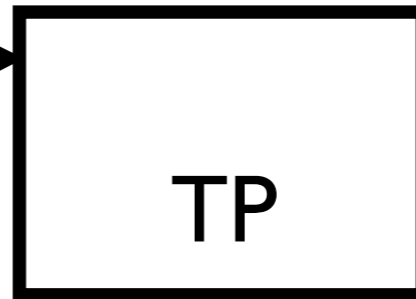
Uşer

# chklibs



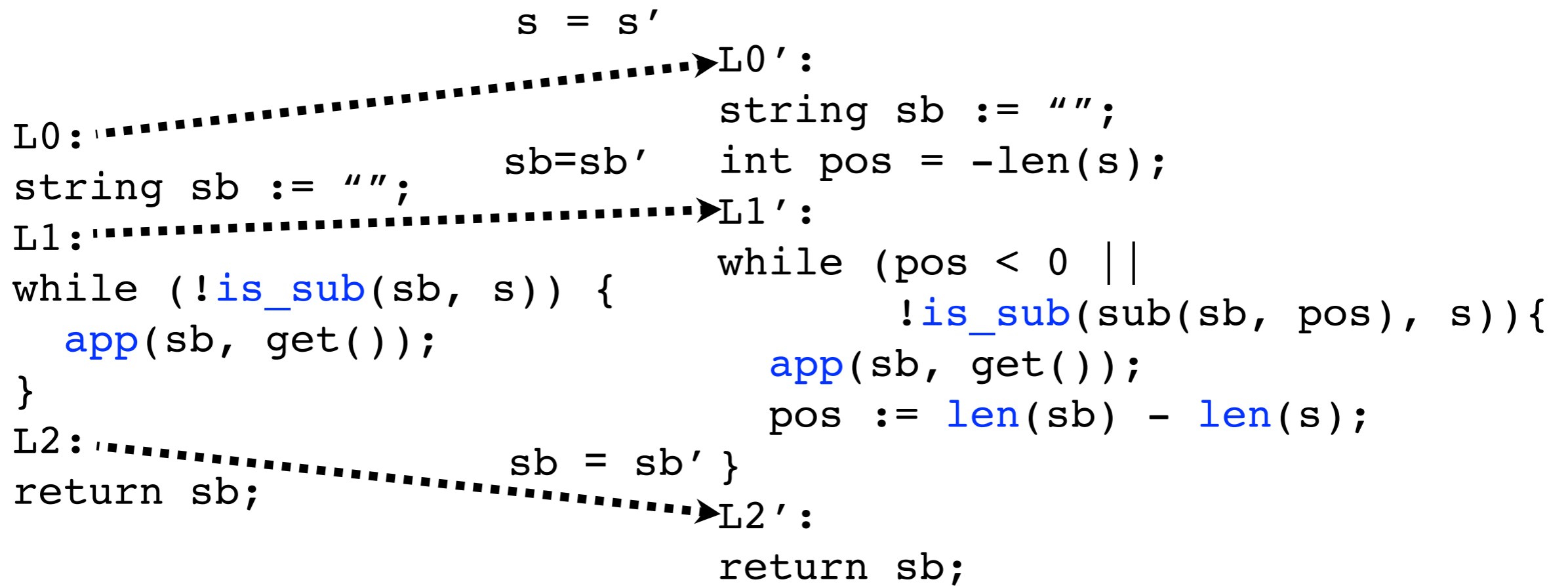
$\text{post} \left[ \begin{array}{l} L1 \rightarrow L2 \\ L1' \rightarrow L2' \end{array} \right] (sb = sb') \Rightarrow sb = sb'$

validated Lib. property: True



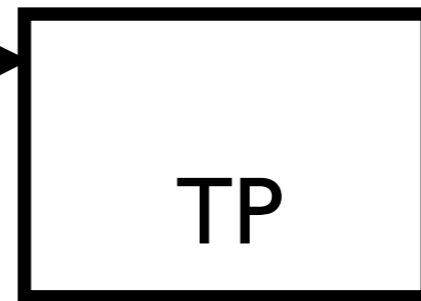
U<sub>9</sub>ser

# chklibs



$\text{post} \left[ \begin{array}{l} L1 \rightarrow L2 \\ L1' \rightarrow L2' \end{array} \right] (sb = sb') \Rightarrow sb = sb'$

validated Lib. property: True



U<sub>ser</sub>

# chklibs

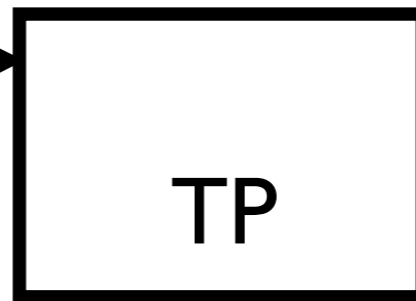
```

                                s = s'
L0: .....>L0':
string sb := "";                string sb := "";
                                int pos = -len(s);
L1: .....>L1':
while (!is_sub(sb, s)) {        while (pos < 0 ||
                                !is_sub(sub(sb, pos), s)) {
    app(sb, get());              app(sb, get());
                                pos := len(sb) - len(s);
L2: .....>L2':
return sb;                       return sb;
                                sb = sb' }

```

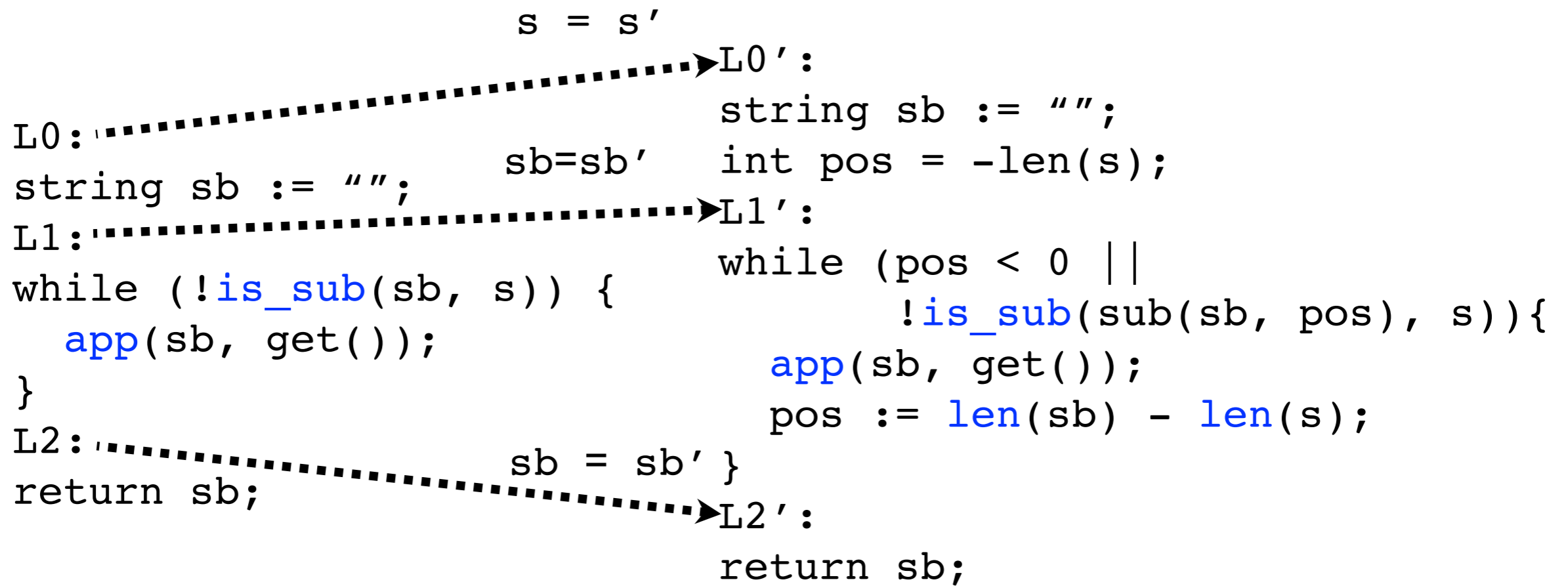
$\text{post} \left[ \begin{array}{l} L1 \rightarrow L2 \\ L1' \rightarrow L2' \end{array} \right] (sb = sb') \Rightarrow sb = sb'$

validated Lib. property: True



User<sub>20</sub>

# chklibs



$\text{post} \left[ \begin{array}{l} L1 \rightarrow L2 \\ L1' \rightarrow L2' \end{array} \right] (sb = sb') \Rightarrow sb = sb'$

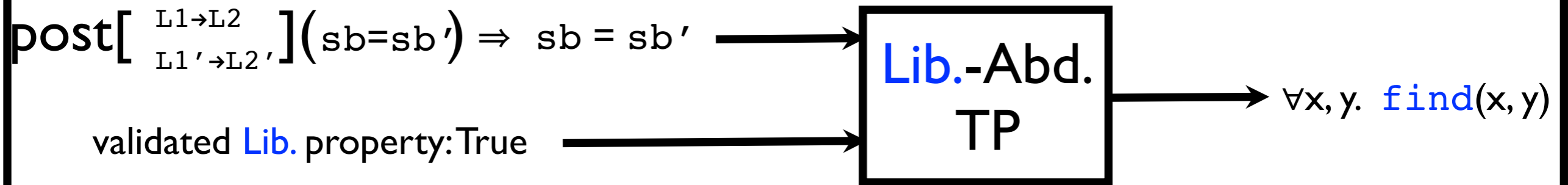
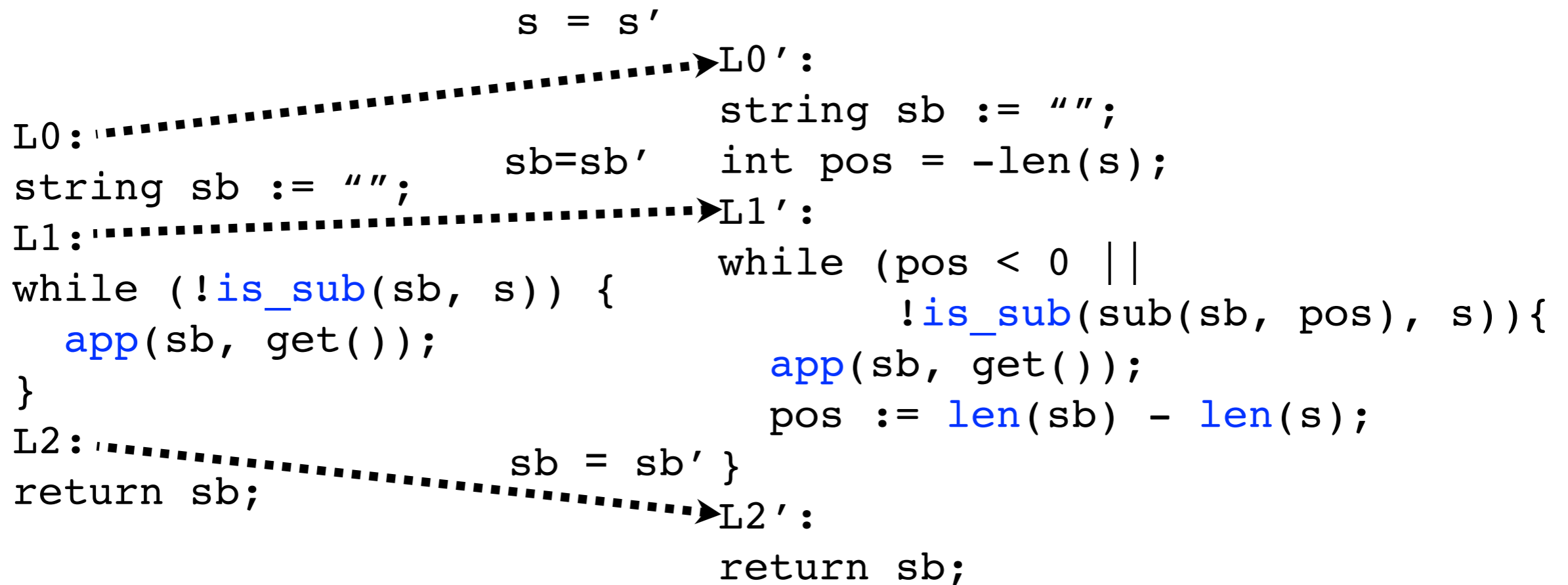
validated Lib. property: True

Lib.-Abd.  
TP

User<sub>20</sub>

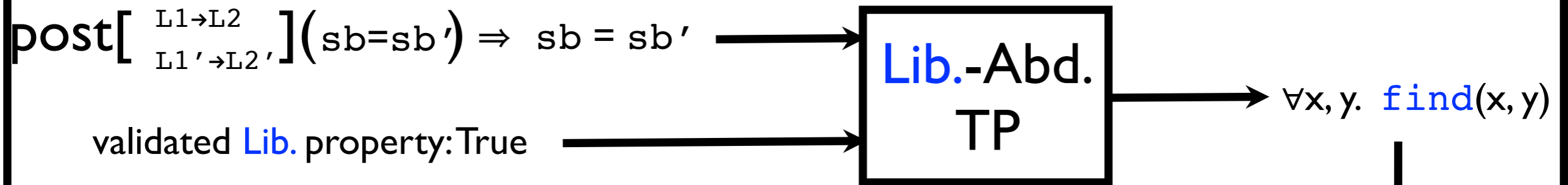
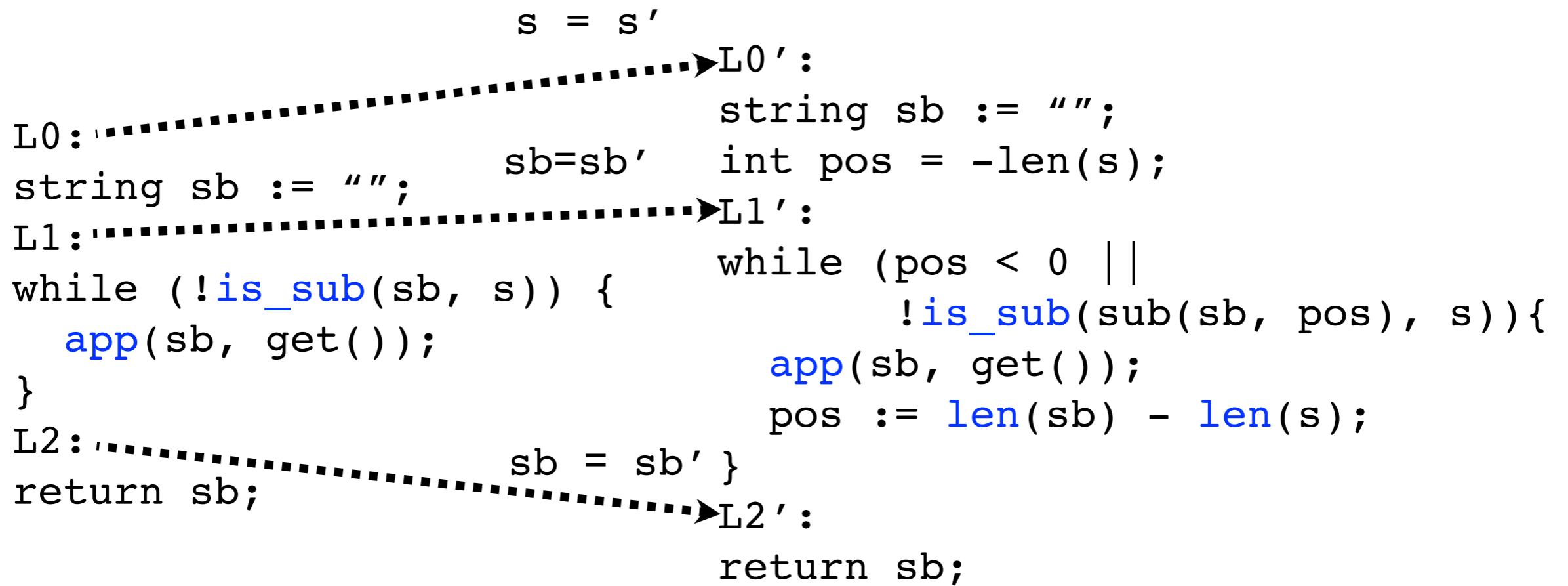


# chklibs



User<sub>20</sub>

# chklibs



User<sub>20</sub>

# Experimental Highlights

- Ran tool on 11 performance-bug patches from Apache, Mozilla, and MySQL
- On average, found supporting conditions
  - in < 1 second
  - with < 10 queries (0 - 7 invalid)

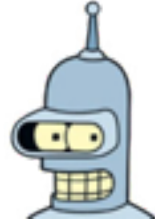
**Thanks  
for your attention!**

# Questions?

```
string sb := "";  
while (!is_sub(sb, s)) {  
    app(sb, get());  
}  
return sb;
```

```
string sb := "";  
int pos = -len(s);  
while (pos < 0 ||  
        !is_sub(sub(sb, pos), s)){  
    app(sb, get());  
    pos := len(sb) - len(s);  
}  
return sb;
```



chklibs 

User 

# Questions?

```
string sb := "";  
while (!is_sub(sb, s)) {  
  app(sb, get());  
}  
return sb;
```

```
string sb := "";  
int pos = -len(s);  
while (pos < 0 ||  
       !is_sub(sub(sb, pos), s)){  
  app(sb, get());  
  pos := len(sb) - len(s);  
}  
return sb;
```

Org.  $\approx$  Opt

assuming

$\forall x, y. \text{len}(x) > \text{len}(y) \Rightarrow$

$\neg \text{is\_sub}(x, y)$

...

chklibs 

$\forall x, y. \text{is\_sub}(x, y)$

$\forall x, y. \text{len}(x) > \text{len}(y) \Rightarrow$

$\neg \text{is\_sub}(x, y)$

Invalid

Valid

...

User 

# Extra Slides

# Apache Bug #19101

## Original

```
vector<BuildListener> ls =  
getBuildListeners();  
for (i=0; i < size(ls); i++) {  
    BuildListener l = at(ls, i);  
    l.startTask();  
}
```

## Optimized



# Apache Bug #19101

## Original

```
vector<BuildListener> ls =  
getBuildListeners();  
for (i=0; i < size(ls); i++) {  
    BuildListener l = at(ls, i);  
    l.startTask();  
}
```

## Optimized

# Apache Bug #19101

## Original

```
vector<BuildListener> ls =  
getBuildListeners();  
for (i=0; i < size(ls); i++) {  
    BuildListener l = at(ls, i);  
    l.startTask();  
}
```

## Optimized

```
int sz =          i  
                sz
```

# Apache Bug #19101

$\forall x, y. \text{size}(\text{at}[\text{arg0}](x, y)) = \text{size}(x)$

## Original

```
vector<BuildListener> ls =  
getBuildListeners();  
for (i=0; i < size(ls); i++) {  
    BuildListener l = at(ls, i);  
    l.startTask();  
}
```

## Optimized

```
int sz =  
i  
sz
```

# I. Non-redundant

```
string sb := "";  
while (!find(sb, s)) {  
    app(sb, get());  
}  
return sb;
```

```
string sb := "";  
int pos = -len(s);  
while (pos < 0  
    !find(sub(sb, pos), s)) {  
    app(sb, get());  
    pos := len(sb) - len(s);  
}  
return sb;
```

The diagram consists of two code snippets at the top, each with a vertical line and an arrow pointing downwards to a large rectangular box. The box contains the text 'chklibs'.

chklibs

The diagram consists of a single large rectangular box containing the text 'User'.

User

# I. Non-redundant

```
string sb := "";  
while (!find(sb, s)) {  
    app(sb, get());  
}  
return sb;
```

```
string sb := "";  
int pos = -len(s);  
while (pos < 0  
        !find(sub(sb, pos), s)) {  
    app(sb, get());  
    pos := len(sb) - len(s);  
}  
return sb;
```

chklibs

$\forall x, y. \text{len}(x) > \text{len}(y) \Rightarrow$

$\neg \text{find}(x, y)$

User

# I. Non-redundant

```
string sb := "";  
while (!find(sb, s)) {  
    app(sb, get());  
}  
return sb;
```

```
string sb := "";  
int pos = -len(s);  
while (pos < 0  
    !find(sub(sb, pos), s)) {  
    app(sb, get());  
    pos := len(sb) - len(s);  
}  
return sb;
```

chklibs

$\forall x, y. \text{len}(x) > \text{len}(y) \Rightarrow$   
 $\neg \text{find}(x, y)$

Valid

User

# I. Non-redundant

```
string sb := "";  
while (!find(sb, s)) {  
    app(sb, get());  
}  
return sb;
```

```
string sb := "";  
int pos = -len(s);  
while (pos < 0  
    !find(sub(sb, pos), s)) {  
    app(sb, get());  
    pos := len(sb) - len(s);  
}  
return sb;
```

chklibs

$\forall x, y. \text{len}(x) > \text{len}(y) \Rightarrow$   
 $\neg \text{find}(x, y)$

$\forall x, y, z. \neg \text{find}(\text{app}(x, y), z) \Rightarrow$   
 $\neg \text{find}(x, z)$

Valid

User

# I. Non-redundant

```
string sb := "";  
while (!find(sb, s)) {  
    app(sb, get());  
}  
return sb;
```

```
string sb := "";  
int pos = -len(s);  
while (pos < 0  
    !find(sub(sb, pos), s)) {  
    app(sb, get());  
    pos := len(sb) - len(s);  
}  
return sb;
```

chklibs

$\forall x, y. \text{len}(x) > \text{len}(y) \Rightarrow$   
 $\neg \text{find}(x, y)$

$\forall x, y, z. \neg \text{find}(\text{app}(x, y), z) \Rightarrow$   
 $\neg \text{find}(x, z)$

Valid

Valid

User



# I. Non-redundant

```
string sb := "";
while (!find(sb, s)) {
  app(sb, get());
}
return sb;
```

```
string sb := "";
int pos = -len(s);
while (pos < 0
      !find(sub(sb, pos), s)) {
  app(sb, get());
  pos := len(sb) - len(s);
}
return sb;
```

chklibs

$$\forall x, y. \text{len}(x) > \text{len}(y) \Rightarrow \neg \text{find}(x, y)$$

$$\forall x, y, z. \neg \text{find}(\text{app}(x, y), z) \Rightarrow \neg \text{find}(x, z)$$

$$\forall x, y, z. \text{len}(\text{app}(x, y)) > \text{len}(z) \Rightarrow \neg \text{find}(x, z)$$

Valid

Valid



User