Review

1. Sample space and Event
   Elementary outcome is an individual possible result of an experiment.
   Sample space **S** = the set of all elementary outcomes.
   An *event* is a collection of elementary outcomes.

   Suppose **A** and **B** are two events:

   (a) **A** OR **B** = the set of elementary outcomes either in **A**, in **B** or in both (union)

   (b) **A** AND **B** = the set of elementary outcomes in both **A** and **B** (intersection)

   (c) NOT **A** = the set of elementary outcomes not in **A** (complement)

   (d) **A** and **B** are mutually exclusive if they do not have elementary outcomes in common.

2. Probabilities are defined in terms of a model. A probability model consists of a probability assignment for each of the events in sample space **S**. Rules for probability assignments:

   (a) For any event **A**, $0 \leq \mathbf{P(A)} \leq 1$

   (b) P(**S**)=1

   (c) If **A** and **B** are mutually exclusive, then P(**A** OR **B**)=P(**A**)+P(**B**)

3. Suppose **A** and **B** are two events

   (a) P(**A** OR **B**)=P(**A**)+P(**B**)-P(**A** AND **B**)

   (b) P(NOT **A**)=1-P(**A**)

   (c) P(**A** AND **B**)=P(**A**)P(**B**) if **A** and **B** are independent

   (d) Conditional probability of event **A** given **B** is

   $$P(\mathbf{A}|\mathbf{B}) = \frac{P(\mathbf{A}\text{ AND }\mathbf{B})}{P(\mathbf{B})}$$

   If **A** and **B** are independent, then $P(\mathbf{A}|\mathbf{B}) = P(\mathbf{A})$.
   So $P(\mathbf{A}\text{ AND }\mathbf{B}) = P(\mathbf{A})P(\mathbf{B})$

   (e) Is there any relationship between mutually exclusive and independence? Answer: No. They are different concepts.

Practice Problem

(a) Suppose that the bowl looks like:

| (1Y) (2Y) (3Y) (1R) (2R) (?R) |

Let **A** be the event that the ball has a 1 on it. Let **B** be the event that the ball is yellow.

i. Suppose $P(\mathbf{B}|\mathbf{A})=\frac{1}{3}$, what is the number corresponding to the ball marked "?"?

ii. Suppose $P(\mathbf{B}|\mathbf{A})=\frac{3}{4}$, is this possible?

(b) 100 students reported the numbers of brothers and sisters they have. The results are summarized in the following table:

		Number of Sisters			
		0	1	≥ 2	total
Number of Brothers	0	3	8	9	20
	1	7	15	8	30
	≥ 2	15	17	18	50
	total	25	40	35	100

Consider the experiment of selecting 1 person at random from a class of 100 students. Compute the following probabilities:

i. Pr(at least 1 sister)

ii. Pr(exactly 1 sister or more than 1 brother)

iii. Pr(exactly 1 brother given that the student has at least 2 sisters)

iv. Pr(exactly 1 sister given that the student has exactly 1 brother)

v. Define A = "having one sister", B = '"having no brothers". Are A and B independent?

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Solutions

(a)

1. $\mathbf{B}$=$\{$1Y, 2Y, 3Y$\}$ and $\mathbf{A}$ AND $\mathbf{B}$=$\{$1Y$\}$,
therefore, $P(\mathbf{B}) = \frac{3}{6} = \frac{1}{2}$ and $P(\mathbf{A}\text{AND}\mathbf{B}) = \frac{1}{6}$.
Now since $P(\mathbf{B}|\mathbf{A}) = \frac{1}{3}$, we have
$$P(\mathbf{A}) = \frac{P(\mathbf{A}\text{AND}\mathbf{B})}{P(\mathbf{B}|\mathbf{A})} = \frac{1/6}{1/3} = \frac{1}{2}.$$
Now we can declare that ? = 1.

2. If $P(\mathbf{B}|\mathbf{A}) = \frac{3}{4}$, then
$$P(\mathbf{A}) = \frac{P(\mathbf{A}\text{AND}\mathbf{B})}{P(\mathbf{B}|\mathbf{A})} = \frac{1/6}{3/4} = \frac{2}{9}.$$
This result is impossible.

(b)

1.

$$\begin{aligned} P(\text{at least 1 sister}) &= P(1S \textit{ or} \geq 2S) \\ &= P(1S) + P(\geq 2S) \\ &= \frac{40}{100} + \frac{35}{100} \\ &= \frac{75}{100} = .75 \end{aligned}$$

2.

$$\begin{aligned} & P(\text{ exactly 1 sister or more than 1 brother}) \\ = \; & P(1Sand0B,\ 1Sand1B,\ 1Sand \geq 2B,\ 0Sand \geq 2B,\ 2Sand \geq 2B) \\ = \; & \frac{8+15+17+15+18}{100} = .73 \end{aligned}$$

3.

$$\begin{aligned} & P(\text{exactly 1 brother given that the student has at least 2 sisters}) \\ = \; & P(1B| \geq 2S) \\ = \; & \frac{P(1Band \geq 2S)}{\geq 2S} \\ = \; & \frac{8/100}{35/100} = 0.2286 \end{aligned}$$

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4.

$$\begin{aligned} & P(\text{exactly 1 sister given that the student has exactly 1 brother}) \\ = & P(1S|1B) \\ = & \frac{P(1Sand1B)}{P(1B)} \\ = & \frac{15/100}{30/100} = .5 \end{aligned}$$

5.

$$P(\mathbf{A}) = P(1S) = \frac{40}{100} = .4,$$

$$P(\mathbf{B}) = P(0B) = \frac{20}{100} = .2,$$

and

$$P(\mathbf{A} \text{ AND } \mathbf{B}) = P(1Sand0B) = \frac{8}{100} = .08.$$

Since $P(\mathbf{A} \text{ AND } \mathbf{B}) = P(\mathbf{A})P(\mathbf{B})$, we can say that **A** and **B** are independent.