Review : Two sample testing and C.I.

1. Paired Sample
   Suppose $(X_1, Y_1), (X_2, Y_2), \cdots, (X_n, Y_n)$ are paired samples from (X, Y) and E(X)= μ_1, E(Y)= μ_2. Suppose $D = X - Y$ is a random sample from a $N(\mu_D, \sigma_D^2)$, where $\mu_D = \mu_1 - \mu_2$. The test statistic for $H_0 : \mu_D = d$ is
   $$T = \frac{\bar{D} - d}{S_D/\sqrt{n}}$$
   with (n-1) degrees of freedom. A $1 - \alpha$ confidence interval for μ_D is given by
   $$\bar{d} - T_{n-1,\alpha/2}\frac{s_d}{\sqrt{n}} \le \mu_D \le \bar{d} + T_{n-1,\alpha/2}\frac{s_d}{\sqrt{n}}$$

2. Independent Sample : $\sigma_1^2 = \sigma_2^2$
   Suppose $X_1, X_2, ..., X_{n_1}$ is a random sample from $N(\mu_1, \sigma_1^2)$ and $Y_1, Y_2, ..., Y_{n_2}$ is a random sample from $N(\mu_2, \sigma_2^2)$. Suppose those two samples are independent and $\sigma_1^2 = \sigma_2^2 = \sigma^2$. The test statistic for $H_0 : \mu_1 - \mu_2 = a$ is
   $$T = \frac{(\bar{X} - \bar{Y}) - a}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}, \quad \text{where} \quad S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$
   with $n_1 + n_2 - 2$ degrees of freedom. A $1 - \alpha$ C.I. for $\mu_1 - \mu_2$ is
   $$\bar{x} - \bar{y} - T_{n_1+n_2-2,\alpha/2}s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \le \mu_1 - \mu_2 \le \bar{x} - \bar{y} + T_{n_1+n_2-2,\alpha/2}s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

3. Independent Sample : $\sigma_1^2 \ne \sigma_2^2$
   Same assumption but variance. The test statistic for $H_0 : \mu_1 - \mu_2 = a$ is
   $$T = \frac{(\bar{X} - \bar{Y}) - a}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}, \quad \text{with} \quad adf = \frac{(vr_1 + vr_2)^2}{(\frac{vr_1^2}{n_1-1}) + (\frac{vr_2^2}{n_2-1})}$$
   where $vr_1 = S_1^2/n_1$ and $vr_2 = S_2^2/n_2$. A $1 - \alpha$ C.I. for $\mu_1 - \mu_2$ is
   $$\bar{x} - \bar{y} - T_{adf,\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \le \mu_1 - \mu_2 \le \bar{x} - \bar{y} + T_{adf,\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

4. Test of equal variance (Levene's Test)
   (a) Determine the median of each sample
   (b) Calculate the absolute value of all deviates from the median
   (c) If, in either sample, there is an odd number of observations, delete exactly one value of "0"
   (d) Perform an independent sample T-test with variances assumed equal

Practice Problem

1. An experiment is conducted to determine if the use of a special chemical additive with a standard fertilizer accelerates plant growth. 10 locations are included in the study. At each location, 2 plants growing in close proximity are treated: one is given the standard fertilizer; the other is given the standard fertilizer with the chemical additive. Plant growth after 4 weeks is measured in cm, and the following data are obtained.

location	1	2	3	4	5	6	7	8	9	10
without additive	20	31	16	22	19	32	25	18	20	19
with additive	23	34	15	21	22	31	29	20	24	23

(a). State the assumptions you must make to proceed with an analysis of data of this form.

(b). Do these data support the claim that use of the chemical additive accelerates plant growth ?

(c). Find a 95% C.I. for the difference between plant growth.

2. Crop rotation seems to change yield in certain situations. Potatoes were gathered from 2 fields, one which had been planted for years with potatoes(X) and one which had previously been planted with corn(Y). The data are:

X	6.5	5.5	5.0	7.0	
Y	7.5	6.5	8.0	9.0	8.5

(a) State the assumptions you must make to proceed with an analysis of data of this form.

(b) Test whether the mean weight of potatoes on the corn field equals to the mean weight of potatoes grown on the non-rotated potato field assuming (i) the variances are equal and (ii) assuming that the variances are not equal.

(c) Check the assumption of equal variance.

(d) Test whether the mean weight of potatoes on the corn field equals to the mean weight of potatoes grown on the non-rotated potato field plus 0.1.

(e) Find a 95% C.I. for the difference in their mean weights.

3. Let $X \sim N(\mu, \sigma^2)$. We wish to test $H_0 : \sigma^2 = 10$ vs $H_a : \sigma^2 > 10$ at $\alpha = 0.1$ with a random sample of size 21. Find the rejection region for this test and the power (approx.) at $\sigma^2 = 15$.