Stat/For/Hort 571 **Yandell/Zhu** **September 11, 2003**

Assignment 2 — Due September 19, 2003

1. I have a loaded die, so some sides have a better chance of coming up than others. Let $X =$ the value that comes up when I roll the die. X can be one of 1, 2, 3, 4, 5, 6. Recall that for a fair die: $P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = 1/6$. Fill in the missing values, if possible. If impossible, say why.

   (a) $P(1) = P(2) = P(3) = P(4) = P(5) = 0.18, P(6) =??$

   (b) $P(1) = P(2) = P(3) = 0.25, P(4) = P(5) = 0.20, P(6) =??$

   (c) $P(1) = 0.1, P(2) = 0.0, P(3) = 0.3, P(4) = 0.1, P(5) = 0.2, P(6) =??$

   (d) $P(1) = 0.5, P(2) = 0.0, P(3) = 0.4, P(4) = 0.1, P(5) = -0.2, P(6) =??$

2. (a) Below I have sketched a bowl with balls in it. Each ball has a number on it. Also, each ball is either red (R) or green (G). I draw a ball at random from the bowl.

   | (1R) (2R) (3R) (3R) (1G) (2G) (3G) (4G) |

   Let A be the event that the ball has a 3 written on it. Let B be the event that the ball is green.

   i. Are A and B independent?

   ii. Are A and B mutually exclusive?

   iii. Find $P(A \text{ OR } B)$.

   (b) Suppose now that the bowl looks like:

   | (1G) (2G) (3G) (3G) (1R) (2R) (3R) (xR) |

   Note that the "last" red ball has a number written on it, but I am not going to tell you what the number is. Let A and B be the same events as in part (a).

   What are the possible values of $P(B|A)$? (Hint: Experiment with various possible number for the last ball. For example, try x = 1, 2, 3, etc.)

3. An entomologist is working with two subspecies of cockroach, German and Asian. Of the Asian subspecies, there are 6 adult males, 7 adult females, and 11 juveniles. Of the German subspecies, there are 4 adult males, 8 adult females and 4 juveniles. Suppose that all 40 cockroaches are placed in a single enclosure. A cockroach is drawn at random from the enclosure; assume that each cockroach has the same chance of being drawn.

   (a) What is the probability that:

   i. the randomly selected cockroach is an adult female?

   ii. the randomly selected cockroach is German and is not adult female?

   iii. the randomly selected cockroach is German given that it is adult female?

   (b) Suppose we have two events defined as follows: Q = "being a German cockroach" and R = "being adult male." Are Q and R independent?

4. In class we have discussed the important result that, if X and Y are two *independent* random variables, then $\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$. The following exercise is concerned with demonstrating the validity of that formula. Recall that the population variance, σ_X^2, of any

random variable, X, is defined as the expected (average) value of the squared deviations from the population mean, μ_X. That is, $\sigma_X^2 = \sum(x - \mu_X)^2 p_X(x)$, in which the sum is over all possible values x that the random variable X can attain and $p_X(x)$ is the probability that $X = x$. Also, the mean μ_X can be written: $\mu_X = \sum x p_X(x)$.

Suppose a bowl contains 100 balls. Each ball has two numbers on it. For 20 of the balls, the first number is 7 and the second number is 4. For another 20 balls, the first number is 7 and the second number is 2. There are also 30 balls with a first number of 1 and a second number of 4. Finally, on the remaining 30 balls, the first number is 1 and the second number is 2. The bowl is sketched below:

20 balls labeled (7|4), 20 balls labeled (7|2), 30 balls labeled (1|4), 30 balls labeled (1|2)

Suppose you draw a ball at random from this bowl. Let X denote the first number on the ball that you draw, and let Y denote the second number on the ball. So, for example, if you draw the ball (7|2) then $X = 7$ and $Y = 2$.

(a) Find $p_X(x)$, and then find the mean and variance of X.
The appropriate formulas are:

$$\mu_X = \sum x p_X(x) \text{ and } \sigma_X^2 = \sum (x - \mu_X)^2 p_X(x)$$

in which the sum is over all possible values x of X and $p_X(x)$ is the probability that $X = x$.

(b) Find $p_Y(y)$, and then find the mean and variance of Y.

(c) Explain why X and Y are independent.

(d) Let W be a new random variable defined by $W = X + Y$. Find $p_W(w)$, and then find the mean and variance of W. Compare $\text{Var}(W)$ with $\text{Var}(X) + \text{Var}(Y)$.

(e) Now consider the new random variable $T = X - Y$. Find $p_T(t)$, and then find the mean and variance of T. Compare $\text{Var}(T)$ with $\text{Var}(X) + \text{Var}(Y)$.

Readings:

- Course Notes: Chapter 3

Notes:

- The blue notes for the course contain a number of minor errors. Errata for these can be found at the course website.