

Solutions for Homework 2

1. (a) $P(6) = 1 - P(1) - P(2) - P(3) - P(4) - P(5) = 0.1$
 (b) $P(1) + P(2) + P(3) + P(4) + P(5) = 1.15 > 1$. Impossible.
 (c) $P(6) = 1 - P(1) - P(2) - P(3) - P(4) - P(5) = 0.3$
 (d) $P(5) < 0$. Impossible.
2. (a) $A = \{3R, 3R, 3G\}$, $B = \{1G, 2G, 3G, 4G\}$, $A \text{ and } B = \{(3G)\}$
 i. $P(A) = 3/8$, $P(B) = 1/2$, $P(A \text{ and } B) = 1/8$
 $P(A)P(B) = (3/8)(1/2) = 3/16 \neq 1/8$. Thus A and B are not independent.
 ii. $P(A \text{ and } B) = 1/8 > 0$. Thus they are not mutually exclusive.
 iii. $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = 3/8 + 1/2 - 1/8 = 3/4$.
- (b) $B = \{1G, 2G, 3G, 3G\}$.
 If $? = 3$, then $A = \{3G, 3G, 3R, 3R\}$ and $P(A) = 1/2$.
 If $? \neq 3$, then $A = \{3G, 3G, 3R\}$ and $P(A) = 3/8$
 $A \text{ and } B = \{3G, 3G\}$, $P(A \text{ and } B) = 1/4$
 If $? = 3$, $P(B|A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{1/4}{1/2} = \frac{1}{2}$;
 If $? \neq 3$, $P(B|A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{1/4}{3/8} = \frac{2}{3}$.

3. (a)

	Male	Female	Juvenile	
German	4	8	4	16
Asian	6	7	11	24
	10	15	15	40

- i. $\frac{15}{40} = 0.375$
 ii. $\frac{4}{40} + \frac{4}{40} = \frac{1}{5} = 0.2$
 iii. $\frac{8/40}{15/40} = \frac{8}{15} = 0.533$
- (b) $P(Q) = \frac{16}{40} = 0.4$, $P(R) = \frac{10}{40} = 0.25$, $P(Q \text{ and } R) = \frac{1}{10} = 0.1$
 $P(Q)P(R) = (0.4)(0.25) = 0.1$. Thus Q and R are independent.
4. (a) $P_X(7) = .4$, $P_X(1) = .6$, $\mu_X = 7(.4) + 1(.6) = 3.4$
 $\sigma_X^2 = (7 - 3.4)^2(.4) + (1 - 3.4)^2(.6) = 8.64$.
- (b) $P_Y(2) = .5$, $P_Y(4) = .5$, $\mu_Y = 3$, $\sigma_Y^2 = 1$.
- (c) $P(X = 7, Y = 4) = P_X(7)P_Y(4) = .2$,
 $P(X = 7, Y = 2) = P_X(7)P_Y(2) = .2$,
 $P(X = 1, Y = 4) = P_X(1)P_Y(4) = .3$,
 $P(X = 1, Y = 2) = P_X(1)P_Y(2) = .3$
- (d) $W = X + Y$
- | | 3 | 5 | 9 | 11 |
|----------|----|----|----|----|
| $P_W(w)$ | .3 | .3 | .2 | .2 |
- $\mu_W = 6.4$, $\sigma_W^2 = 9.64 = \sigma_X^2 + \sigma_Y^2$.
- (e) $T = X - Y$
- | | -3 | -1 | 3 | 5 |
|----------|----|----|----|----|
| $P_T(t)$ | .3 | .3 | .2 | .2 |
- $\mu_W = .4$, $\sigma_W^2 = 9.64 = \sigma_X^2 + \sigma_Y^2$.