STAT 571, Solution for Assignment #3

September 22, 2003

2. X = # of diseased horses needing isolation room

a.

$$P(X=1) = \frac{6!}{1!5!}(0.20)^1(0.80)^5 = 0.3932.$$

b.

$$\begin{aligned} & P(X \le 2) \\ &= \quad P(X=0) + P(X=1) + P(X=2) \\ &= \quad 0.1678 + 0.3355 + 0.2936 \\ &= \quad 0.7969 \end{aligned}$$

3. a.

$$P(X=0) \;=\; P\{TTT\} = 0.5(0.6)(0.6) = 0.18.$$

$$\begin{aligned} P(X=1) &= P\{HTT, THT, TTH\} \\ &= P\{HTT\} + P\{THT\} + P\{TTH\} \\ &= 0.5(0.4)(0.6) + 0.5(0.6)(0.4) + 0.5(0.4)(0.4) \\ &= 0.32 \end{aligned}$$

By symmetry, we have

$$P(X=2) = P(X=1) = 0.32,$$

$$P(X=3) = P(X=0) = 0.18.$$

b.

$$\begin{aligned} P\{H1st\} &= P\{HHH\} + P\{HTH\} + P\{HHT\} + P\{HTT\} \\ &= 0.5(0.6)(0.6) + 0.5(0.6)(0.4) + 0.5(0.4)(0.4) + 0.5(0.4)(0.6) \\ &= 0.5. \end{aligned}$$

similarly, $P\{H2nd\} = P\{H3rd\} = 0.5$.

c. 1.

$$P(X = 0) = P\{TTT\} = 0.5(0.9)(0.9) = 0.405.$$

By symmetry, we have $P(X = 3) = P(X = 0) = 0.405$.

$$P(X = 2) = P(X = 1) = \frac{1 - 2(0.405)}{2} = 0.095.$$

2.

$$\begin{aligned} P\{H1st\} &= P\{HHH\} + P\{HTH\} + P\{HTT\} + P\{HTT\} \\ &= 0.5(0.9)(0.9) + 0.5(0.1)(0.1) + 0.5(0.1)(0.9) + 0.5)0.1)(0.9) \\ &= 0.5. \end{aligned}$$

Or, we can get this result simplily by summetry.
Similarly, $P\{H2nd\} = P\{H3rd\} = 0.5$.

d. Two assumptions are violated:
1. Each trial is not independent;
2. The probability of success is not the same.

4. a. i.

$$P(Z \leq 0.65) = 1 - P(Z > 0.65) = 1 - 0.2578 = 0.7422.$$

ii.

$$P(Z \geq -1.32.) = 1 - P(Z > 1.32) = 1 - 0.0934 = 0.9066.$$

iii.

$$\begin{aligned} P(-1.25 \leq Z \leq 0.58) &= P(Z \leq 0.58) - P(Z < -0.1.25) \\ &= 1 - P(Z > 0.58) - P(Z > 1.25) \\ &= 1 - 0.2810 - 0.1056 \\ &= 0.6134. \end{aligned}$$

iv.

$$\begin{aligned} P(1.4 \leq Z \leq 2.4) &= P(Z \geq 1.4) - P(Z > 2.4) \\ &= 0.0808 - 0.0082 = 0.0726. \end{aligned}$$

v.

$$\begin{aligned} P(-1.5 \leq Z \leq -1.2) &= P(1.2 \leq Z \leq 1.5) \\ &= P(Z \geq 1.2) - P(Z > 1.5) \\ &= 0.1151 - 0.0668 = 0.0483. \end{aligned}$$

b. i.

$$P(-0.639 \leq Z) = P(Z \leq 0.639) = 0.7386.$$

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> pnorm(.639,0,1) = .7386
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ii.

$$\begin{aligned} P(0.427 \leq Z \leq 1.295) &= P(Z \leq 1.295) - P(Z < 0.427) \\ &= 0.9023 - 0.6653 = 0.237. \end{aligned}$$

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>pnorm(1.295,0,1) - pnorm(.427,0,1) = .237
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5. a. i.

$$P(X \leq a) = P(X \geq -a) = 0.3.$$

From the table, we get $-0.53 < a < -0.52$.

$$P(X \geq -b) = P(X \leq b) = 0.2.$$

From the table, we have $-0.85 < b < -0.84$.

ii.

$$P(X > a) = 0.25 = P(\frac{X-10}{6} > \frac{a-10}{6}),$$

then $0.67 < \frac{a-10}{6} < 0.68$, then $14.02 < a < 14.08$.

$$P(X \geq b) = 0.15 = P(\frac{X-10}{6} \geq \frac{b-10}{6}),$$

then $1.03 < \frac{b-10}{6} < 1.04$, then $16.18 < b < 16.24$.

iii.

$$P(X \leq 0) = P(X + 32 \leq 32) = P(Z \leq 1.5) = 1 - 0.0668 = 0.9332.$$

$$P(X \geq b) = 0.01 = P(\frac{X+3}{2} \geq \frac{b+3}{2}) = P(Z \geq \frac{b+3}{2}).$$

Then $2.32 < \frac{b+3}{2} < 2.33$, then $1.64 < b < 1.66$.

iv.

$$30.4 < a < 30.425, P(X \geq 31.81) = 0.6844.$$

v.

$$4.65 < \sigma < 4.6875.$$

b.

$$P(X \geq -5.7) = 1 - 0.2954 = 0.7064.$$

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> 1 - pnorm(-5.7,-4,sqrt(10)) = .7064
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c.

$$P(-2.0 \leq X \leq 8.6) = 0.5764.$$

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> pnorm(8.6,-4,sqrt(10)) - pnorm(-2.0,-4,sqrt(10)) = .5764
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d.

$$x^* = 103.3707.$$

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> qnorm(.3,104,sqrt(1.44)) = 103.3707
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6. a.

$$P(0.65 < X < 0.9) = 0.9342.$$

b.

$$P(X \geq .7840) = .6$$

c.

$$0.6760 < x < 0.9240.$$