Solution for Assignment 7

1. (a)
```
> draws=matrix(rnorm(200*15,62,9),15)
> get.conf.int=function(x) t.test(x)$conf.int
> conf.int=apply(draws,2,get.conf.int)
> plot(range(conf.int),c(0,201),type="n",xlab="confidence interval for mean",
+   ylab="sample run")
> for(i in 1:200) lines(conf.int[,i],rep(i,2))
> abline(v=62, lwd=2,lty=2)
> sum(conf.int[1,]<=62 & conf.int[2,]>=62)
[1] 191
```
   (b) About the same proportion since $1-\alpha$=95% C.I. covers 62 if and only if you do not reject $H_0 : \mu = 62$ at $\alpha = 0.05$.

2. (a)
```
> mydata=matrix(rnorm(3*10,75,10),10)
> par(mfrow=c(1,3))
> for (i in 1:3) qqnorm(mydata[,i])
```
   (b)
```
> mydata=matrix(rnorm(3*40,75,10),40)
> par(mfrow=c(1,3))
> for (i in 1:3) qqnorm(mydata[,i])
```
   (c) From the plots, we can see that for small sample size it is not easy to detect normality from the stem-and-leaf display. Normal score plot is better for detecting normality.

   And the normal score plot fits a straight line better for large sample size($n = 40$) than samll sample size($n = 10$).

3. From the normal score plots, we find the one for the log units appears more normal. So we contruct 0.95 t-interval for log units with base e

$$\begin{aligned}\bar{x} \pm t_{0.025} s/\sqrt{12} &= 15.332 \pm 2.201 \times 0.918/\sqrt{12} \\ &= (14.749, 15.915)\end{aligned}$$

With base 10

$$\begin{aligned}\bar{x} \pm t_{0.025} s/\sqrt{12} &= 6.659 \pm 2.201 \times 0.399/\sqrt{12} \\ &= (6.405, 6.913)\end{aligned}$$

4.

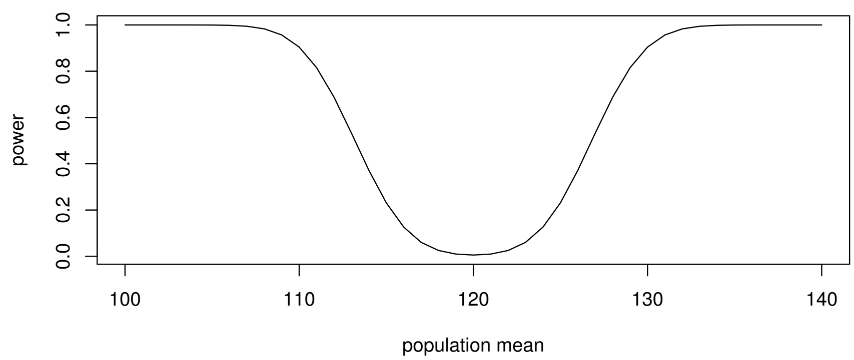
$$\begin{aligned}\text{power(when } \mu = \mu_0) &= P(\text{reject } H_0 | \mu = \mu_0) \\ &= P(\bar{X} \le 113.2 \text{ or } \bar{X} \ge 126.8 | \mu = \mu_0) \\ &= P(Z \le \frac{113.2 - \mu_0}{12/\sqrt{24}}) + P(Z \ge \frac{126.8 - \mu_0}{12/\sqrt{24}})\end{aligned}$$

Then you can obtain the following powers:

mu	108	112	116	120	124	128	132
power	0.9831	0.6879	0.1265	0.0055	0.1265	0.6879	0.9831

The power at μ=120 means α or type I error probability.

```
> mus=seq(100,140,by=1)
> plot(mus,pnorm(113.2,mus,12/sqrt(24))+pnorm(126.8,mus,12/sqrt(24),lower.tail=F),
+     type='l',xlab="population mean",ylab="power")
```



5. This is to do test:

$$H_0\text{: } \mu \leq 15.0 \text{ vs. } H_A\text{: } \mu > 15.0.$$

(a) Suppose the rejection criterion is that $\bar{X} \geq c$ and the required sample size is n, then we have
$\alpha = P(\text{reject } H_0|H_0)$=$P(\bar{X} \geq c|\mu = 15)$= $P(Z \geq \frac{c-15}{3.8/\sqrt{n}})$
power=1-β=$P(\text{reject } H_0|H_A \text{ at } \mu = 17.2)$=$P(\bar{X} \geq c|\mu = 17.2)$= $P(Z \geq \frac{c-17.2}{3.8/\sqrt{n}})$
From above we have the following two equations:

$$\frac{c-15}{3.8/\sqrt{n}} = Z_\alpha$$

$$\frac{c-17.2}{3.8/\sqrt{n}} = -Z_\beta$$

For (a), plug in $\alpha = 0.025$ and $\beta = 0.1$, solve these two equations for c and n, we have $n = 31.3 \approx 32$, $c = 16.33$.

(b) For (b), plug in $\alpha = 0.10$ and $\beta = 0.1$, solve those two equations for c and n, we have $n = 19.6 \approx 20$, $c = 16.1$.

(c) For (c), plug in $\alpha = 0.025$ and $\beta = 0.25$, solve those two equations for c and n, we have $n = 20.7 \approx 21$, $c = 16.64$.

(d) Smaller α or larger power will require a larger sample size.