Reinforcement Learning
Part 2

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[Based on slides from David Page, Mark Craven]

Goals for the lecture

you should understand the following concepts

- value functions and value iteration (review)
- Q functions and Q learning
- exploration vs. exploitation tradeoff
- compact representations of Q functions

Value function for a policy

- given a policy $\pi : S \rightarrow A$ define

$$V^{\pi}(s) = \sum_{t=0}^{\infty} \gamma^t E[r_t]$$

assuming action sequence chosen according to π starting at state s

- we want the optimal policy π^* where

$$p^* = \arg\max_p V^p(s) \quad \text{for all } s$$

we'll denote the value function for this optimal policy as $V^*(s)$

Value iteration for learning $V^*(s)$

initialize $V(s)$ arbitrarily

loop until policy good enough

{

  loop for $s \in S$

  {

    loop for $a \in A$

    {

$$Q(s,a) \leftarrow r(s,a) + \gamma \sum_{s' \in S} P(s'|s,a) V(s')$$

    }

$$V(s) \leftarrow \max_a Q(s,a)$$

  }

}

Q functions

define a new function, closely related to V^*

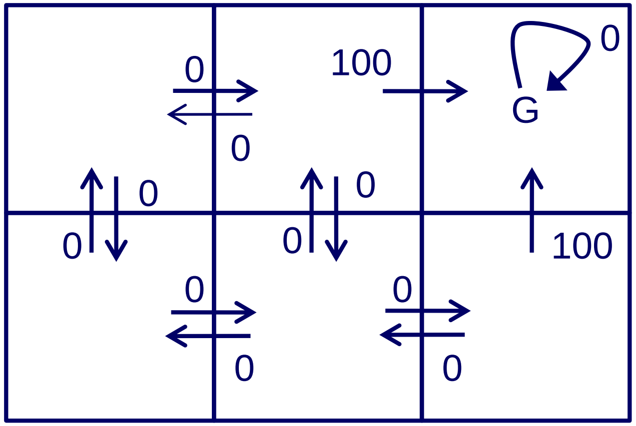
$$Q(s,a) \leftarrow E\left[r(s,a)\right] + \gamma E_{s'|s,a}\left[V^*(s')\right]$$

if agent knows $Q(s, a)$, it can choose optimal action without knowing $P(s' \mid s, a)$

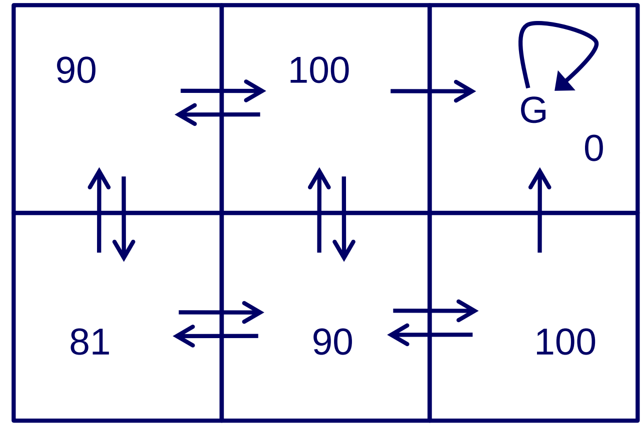
$$\pi^*(s) \leftarrow \arg\max_a Q(s,a) \qquad V^*(s) \leftarrow \max_a Q(s,a)$$

and it can learn $Q(s, a)$ without knowing $P(s' \mid s, a)$

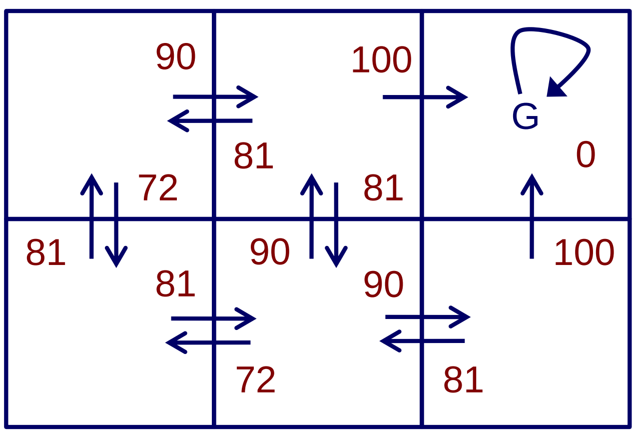
Q values



$r(s, a)$ (immediate reward) values



$V^*(s)$ values



$Q(s, a)$ values

Q learning for deterministic worlds

for each s, a initialize table entry $\hat{Q}(s,a) \leftarrow 0$

observe current state s

do forever

  select an action a and execute it

  receive immediate reward r

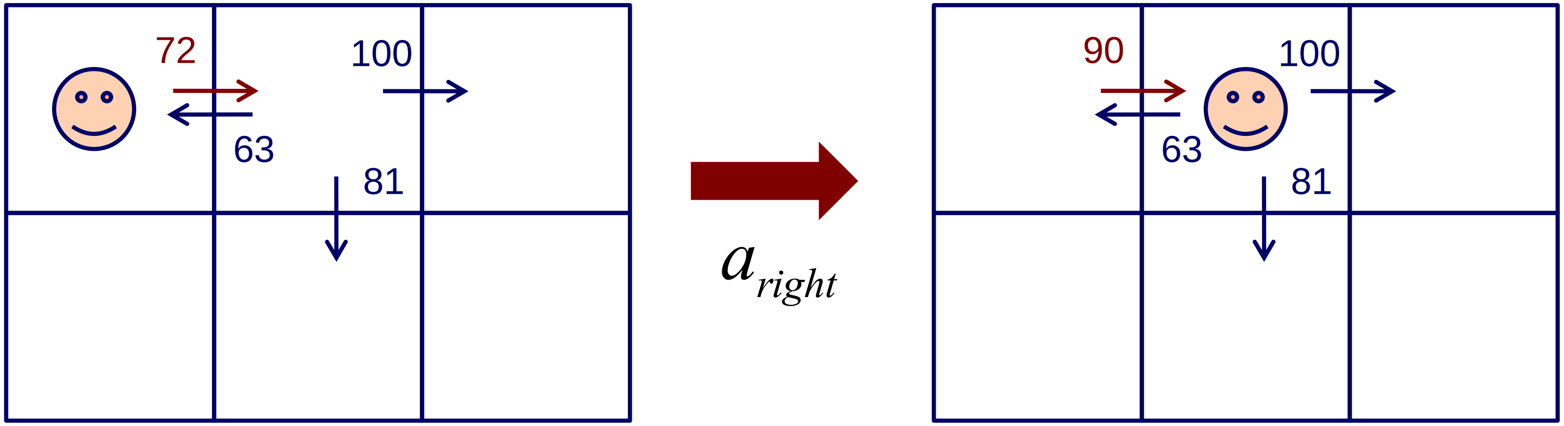
  observe the new state s'

  update table entry

  $$\hat{Q}(s,a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s',a')$$

  $s \leftarrow s'$

Updating Q

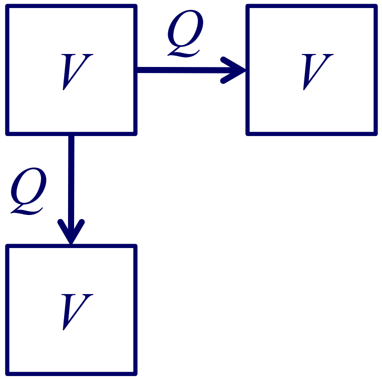


$$\hat{Q}(s_1, a_{right}) \leftarrow r + \gamma \max_{a'} \hat{Q}(s_2, a')$$

$$\leftarrow 0 + 0.9 \max\{63, 81, 100\}$$

$$\leftarrow 90$$

Q's vs. V's



- Which action do we choose when we're in a given state?
- V's (model-based)
  - need to have a 'next state' function to generate all possible states
  - choose next state with highest V value.
- Q's (model-free)
  - need only know which actions are legal
  - generally choose next state with highest Q value.

Exploration vs. Exploitation

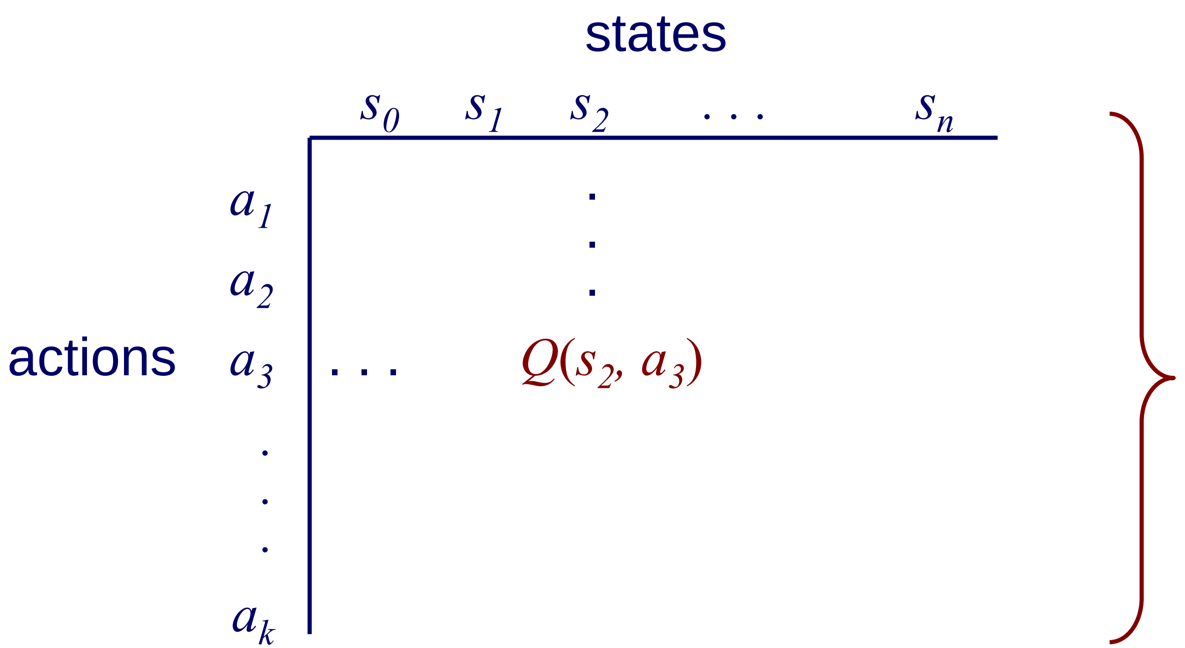
- in order to learn about better alternatives, we shouldn't always follow the current policy (exploitation)
- sometimes, we should select random actions (exploration)
- one way to do this: select actions probabilistically according to:

$$P(a_i \mid s) = \frac{c^{\hat{Q}(s,a_i)}}{\sum_j c^{\hat{Q}(s,a_j)}}$$

where $c > 0$ is a constant that determines how strongly selection favors actions with higher Q values

Q learning with a table

As described so far, Q learning entails filling in a huge table

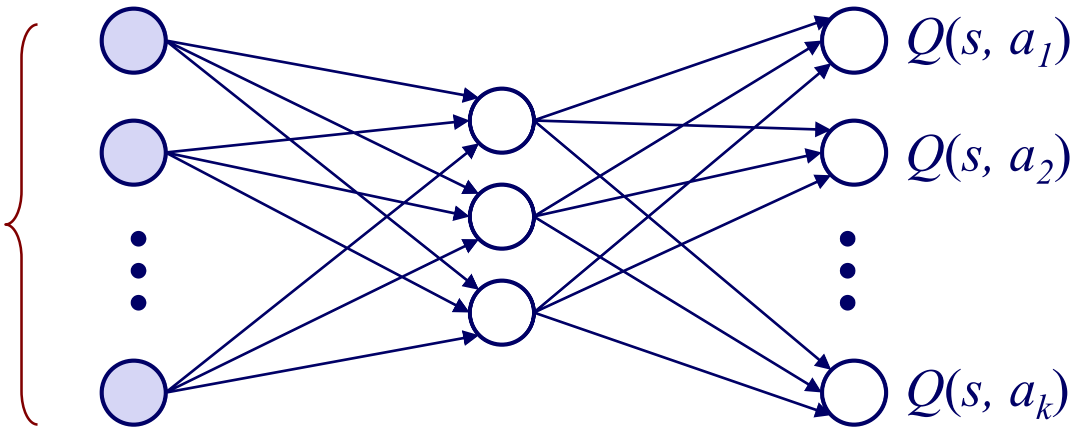


A table is a very <u>verbose way to</u> represent a function

Representing Q functions more compactly

We can use some other function representation (e.g. a neural net) to compactly encode a substitute for the big table

encoding of the state (s)



each input unit encodes a property of the state (e.g., a sensor value)

or could have one net for each possible action

Why use a compact Q function?

1. Full Q table may not fit in memory for realistic problems
2. Can generalize across states, thereby speeding up convergence

   i.e. one instance 'fills' <u>many</u> cells in the Q table

<u>Notes</u>

1. When generalizing across states, cannot use $\alpha=1$
2. Convergence proofs only apply to Q <u>tables</u>
3. Some work on bounding errors caused by using compact representations (e.g. Singh & Yee, *Machine Learning* 1994)

Q tables vs. Q nets

Given: 100 Boolean-valued features

10 possible actions

Size of Q table

10×2^{100} entries

Size of Q net (assume 100 hidden units)

$$\underbrace{100 \times 100}_{\text{weights between inputs and HU's}} + \underbrace{100 \times 10}_{\text{weights between HU's and outputs}} = 11{,}000 \text{ weights}$$

Representing Q functions more compactly

- we can use other regression methods to represent Q functions
  - k-NN
  - regression trees
  - support vector regression
  - etc.

Q learning with function approximation

1. measure sensors, sense state s_0
2. predict $\hat{Q}_n(s_0, a)$ for each action a
3. select action a to take (with randomization to ensure exploration)
4. apply action a in the real world
5. sense new state s_1 and immediate reward r
6. calculate action a' that maximizes $\hat{Q}_n(s_1, a')$
7. train with new instance

$$\boldsymbol{x} = s_0$$

$$y \leftarrow (1-\alpha)\hat{Q}(s_0, a) + \alpha\left[r + \gamma \max_{a'} \hat{Q}(s_1, a')\right]$$

Calculate Q-value you would have put into Q-table, and use it as the training label