(Sample Solutions)
Midterm Examination CS540-1: Introduction to Artificial Intelligence Fall 2016
(10 questions, 10 points each)

1. A search tree has 4 levels (the root is at level 1), and every internal node points to 3 children. Suppose there is no goal node. How many goal checks will breadth first search perform?

   The number of nodes at each level is: 1, 3, 9, 27. Each needs to be goal checked. The total is 40.
   5 points for: simple arithmetic error, or off by one level (i.e. thought there were one more level of 81 leaves).

2. Consider the following graph with n nodes: node 1 points to node 2, 2 points to 3, 3 to 4, $\ldots$, node $n-1$ to n, and node 1 also points to node n. There is no goal node. Start from node 1 and perform path check DFS, how many nodes will be goal checked? Assume we always push successors into stack with smaller node number first. (Clarification: this is a directed graph.)

   The stack for the first few steps is:
   (bottom) 1
   (bottom) 2, n
   (bottom) 2
   (bottom) 3
   ...
   Eventually we pop oout $n-1$ and will push in n, since path check DFS doesn't see n on the stack at that moment. Node n is checked twice, the other nodes onces, so the total is $n+1$.
   5 points for the answer n if the student articulate that n has been visited before (which is a confusion with memorizing DFS).

3. Write down the formula for

$$db + (d-1)b^2 + (d-2)b^3 + \ldots + b^d$$

Let

$$S = db + (d-1)b^2 + (d-2)b^3 + \ldots + b^d$$

Then

$$bS = db^2 + (d-1)b^3 + (d-2)b^4 + \ldots + b^{d+1}$$

Subtracting, we have

$$bS - S = -db + b^2 + b^3 + \ldots + b^d + b^{d+1}.$$

Now let

$$T = b^2 + b^3 + \ldots + b^d + b^{d+1}.$$

Applying the same trick,

$$T/b = b^1 + b^2 + \ldots + b^{d-1} + b^d.$$

$$T - T/b = b^{d+1} - b.$$

$$T = \frac{b^{d+2} - b^2}{b-1}.$$

Thus

$$bS - S = -db + T = -db + \frac{b^{d+2} - b^2}{b-1}$$

Finally

$$S = -\frac{db}{b-1} + \frac{b^{d+2} - b^2}{(b-1)^2}.$$

5 points for demonstrating the idea of series sum (e.g. multiplying by b and subtracting), but tragic loss of sanity later.
Unfortunately, no points for answers like $\sum_{i=1}^{d}(d-i-1)b^i$.

4. Assume h and h' are any admissible heuristic functions. Consider any real number a and the new heuristic $h''(s) = ah(s) + (1-a)h'(s)$. For what range of a is h'' guaranteed to be a heuristic function? (Clarification: h'' must be admissible.)

The only a that will guarantee h'' to be admissible for any admissible h and h' is in the interval $[0, 1]$.
5 points for $(0, 1)$, or $a \leq 1$.

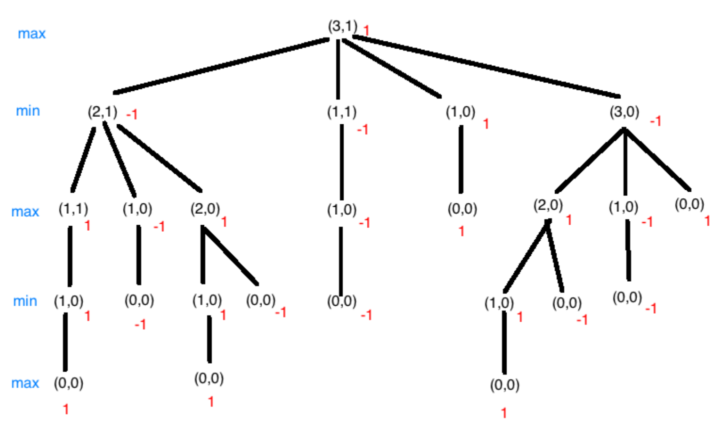
5. You have a fair coin (equal probability). Flipping it n times and they all happen to be heads. What is the probability that the next three flips will contain one or more tails?

   Of course the first n outcomes are independent of the rest, since we know it's a fair coin. The event is the complement of the event that all three flips come up heads. The latter has probability $(1/2)^3$. So the desired probability is $1 - (1/2)^3 = 7/8 = 0.875$.
   7 points for minor calculation errors.
   3 points for explaining 'flips are independent of each other' and answer of 0.5.

6. Consider the following version of hill climbing: at initial state s we randomly choose one of s's neighbors with equal probability. If the chosen neighbor has a strictly better score than s we move to the neighbor; otherwise we stay at s. Assume s has n neighbors, and only one of the n neighbors has a strictly better score than s. What is the chance that we move out of s in T iterations or less?

   The event is the complement of the event that all T iterations were unlucky where we missed the best neighbor. The probability of the latter is $(1 - 1/n)^T$. Therefore the answer is $1 - (1 - 1/n)^T$.
   5 points for 1 - P(not moving in T iterations) $= (1 - 1/n)^T$.
   3 points for P(not moving in one iteration) $= (1 - 1/n)$.

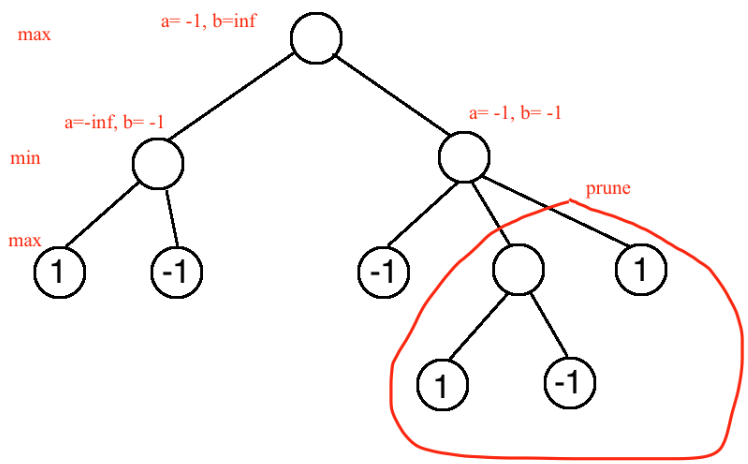
7. Consider the 2-nim game where the first pile has 3 sticks and the second pile has 1 stick, i.e. the initial state is (3,1). A player must take part or all of a pile in each move. The player who takes the last stick loses. Draw the game tree and attach game theoretical values to each node.



It is OK if your leaf nodes are at $(1, 0)$ instead of $(0, 0)$, or if you distinguish states (a, b) and (b, a).

5 points for complete and correct game tree + 5 points for complete and correct game theoretical values.
7 points for correct game tree, minor errors in game values.
5 points for correct game tree, incorrect game values.
3 points for partly correct game tree.

8. Perform alpha-beta pruning on the following game tree. Max moves first, Min moves second.



Correctly pruned game tree but no alpha, beta values are written down: 7 points
Correct alpha and beta values but not pruned the tree: 7 points
Correct alpha and beta values but pruning occurs later than expected: 5 points

9. Enumerate the pure strategies for each player in the previous question. Write down the corresponding matrix normal form of the game. (Clarification: This question does not involving pruning.)

Let the four internal nodes be A, B, C, D in the usual order. Max's pure strategies are:
A-I: A:L, D:L
A-II: A:L, D:R
A-III: A:R, D:L
A-IV: A:R, D:R
Min's pure strategies are: B-I: B:L, C:L
B-II: B:L, C:C
B-III: B:L, C:R
B-IV: B:R, C:L
B-V: B:R, C:C
B-VI: B:R, C:R

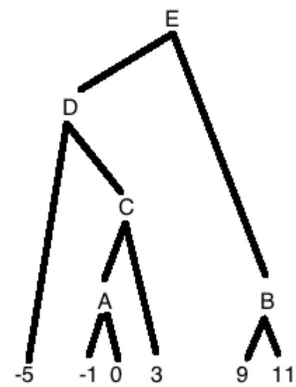
The matrix normal form is

	B-I	B-II	B-III	B-IV	B-V	B-VI
A-I	1	1	1	-1	-1	-1
A-II	1	1	1	-1	-1	-1
A-III	-1	1	1	-1	1	1
A-IV	-1	-1	1	-1	-1	1

Missing matrix normal form, but correct enumeration: 5 points
Missing enumeration but have matrix normal form (with unknown notation): 7 points if the values are same as the answer; 0 otherwise

10. Perform hierarchical clustering with single linkage in one-dimensional space on the following points: -5, -1, 0, 3, 9, 11. Draw the clustering tree.

Iteration 1: group A=(-1, 0): distance 1
Iteration 2: group B=(9, 11): distance 2
Iteration 3: group C=(A, 3): distance 3 between 3 and the element 0 in A
Iteration 4: group D=(-5, C): distance 4 between -5 and the element -1 in C
Iteration 5: group E=(D, B): distance 6 between elements 3 and 9.



No tree graph, but correct steps: 7 points
Almost correct graph and solution, minor errors: 7 points