CS 540: Introduction to Artificial Intelligence
Homework Assignment # 5

Assigned: 3/5
Due: 3/12 before class

Hand in your homework:

If a homework has programming questions, please hand in the Java program. If a homework has written questions, please hand in a PDF file. Regardless, please zip all your files into hwX.zip where X is the homework number. Go to UW Canvas, choose your CS540 course, choose Assignment, click on Homework X: this is where you submit your zip file.

Late Policy:

All assignments are due at the beginning of class on the due date. One (1) day late, defined as a 24-hour period from the deadline (weekday or weekend), will result in 10% of the total points for the assignment deducted. So, for example, if a 100-point assignment is due on a Wednesday 9:30 a.m., and it is handed in between Wednesday 9:30 a.m. and Thursday 9:30 a.m., 10 points will be deducted. Two (2) days late, 25% off; three (3) days late, 50% off. No homework can be turned in more than three (3) days late. Written questions and program submission have the same deadline.

Assignment grading questions must be raised with the instructor within one week after the assignment is returned.

Collaboration Policy:

You are to complete this assignment individually. However, you are encouraged to discuss the general algorithms and ideas with classmates, TAs, and instructor in order to help you answer the questions. You are also welcome to give each other examples that are not on the assignment in order to demonstrate how to solve problems. But we require you to:

- not explicitly tell each other the answers
- not to copy answers or code fragments from anyone or anywhere
- not to allow your answers to be copied
- not to get any code on the Web

In those cases where you work with one or more other people on the general discussion of the assignment and surrounding topics, we suggest that you specifically record on the assignment the names of the people you were in discussion with.

Question 1: Probability [30 points]

1. (10 points) Assume that we are given a set of probability tables with or without independence assumptions. Derive an expression for the requested probability tables in terms of the given probability tables. If there is no such expression, explain why not.

   (a) Given probability tables $P(Y)$, $P(Z)$, $P(X|Y)$, $P(Z|X,Y)$ without any independence assumptions, derive an expression to calculate the table $P(X,Y|Z)$.

   (b) Given probability tables $P(Y)$, $P(X|Y,Z)$, $P(Y|X,Z)$, $P(Z|X,Y)$ with a conditional independence assumption $X \perp\!\!\!\perp Y|Z$, derive an expression to calculate the table $P(X,Y,Z)$.

   (c) Given probability tables $P(X)$, $P(Y|X)$, $P(Z|Y)$, $P(X|Y,Z)$ with an independence assumption $X \perp\!\!\!\perp Y$, derive an expression to calculate the table $P(Z)$.

   (d) Given probability tables $P(X)$, $P(Y|X)$, $P(X|Y,Z)$, $P(Z|X,Y)$ without any independence assumptions, derive an expression to calculate the table $P(Y|X,Z)$.

2. (10 points) For each of the following questions, what is the minimal set of independence assumptions needed in order for the equation to be true.

   Choose from the following independence assumptions $X \perp\!\!\!\perp Y, X \perp\!\!\!\perp Z, Y \perp\!\!\!\perp Z, X \perp\!\!\!\perp Y|Z, X \perp\!\!\!\perp Z|Y, Y \perp\!\!\!\perp Z|X$ and briefly explain your reasons. If no independence assumptions are needed, why not.

   (a) $P(X,Y) = P(X|Z)P(Y)$

   (b) $P(X|Y) = \frac{P(Y|X)P(X)}{P(Y)}$

   (c) $P(Z|X,Y) = \frac{P(X|Z)P(Y|Z)P(Z)}{P(X|Y)P(Y)}$

   (d) $P(X,Y) = \sum_{z \in Z} P(X|Y)P(Y|z)P(z)$

3. For each of the following questions, select **all** expressions that are equal to:

   (a) (5 points) $P(X|Y,Z)$ without any independence assumptions. Briefly explain your reasons.

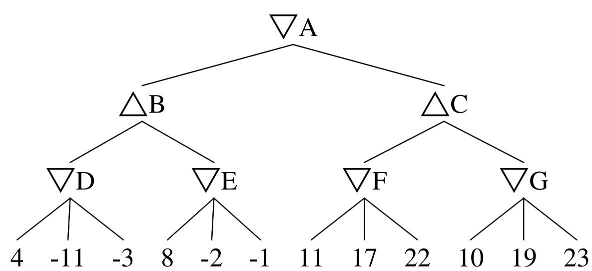
   - $\frac{\sum_{w \in W} P(X,Y,Z,w)}{\sum_{w \in W} P(Y,Z,w)}$
   - $\frac{P(X|Z)P(Y|X)}{\sum_{w \in W} P(Y|Z,w)}$
   - $\frac{P(X|Y,Z)}{P(Y|Z)}$
   - $\sum_{w \in W} P(X|Y,Z,w)$
   - $\sum_{w \in W} P(X,w|Y,Z)$

   (b) (5 points) $P(X|Z)$ with a conditional independence assumption $X \perp\!\!\!\perp Y|Z$. Briefly explain your reasons.

- $\frac{P(X,Y|Z)}{P(Y|Z)}$
- $P(X|Y)P(Y|Z)$
- $\frac{P(X,Y,Z)}{P(Y,Z)P(Y)}$
- $\frac{\sum_{y \in Y} P(X,y|Z)P(Z)}{P(Z)}$
- $\frac{P(X,Z|Y)P(Y)}{P(Z)P(Y|Z)}$

Question 2: Game Tree Search [20 points]

Consider the zero-sum game tree below. Denote $\triangle$ and $\bigtriangledown$ nodes as the maximizing and minimizing players, respectively.



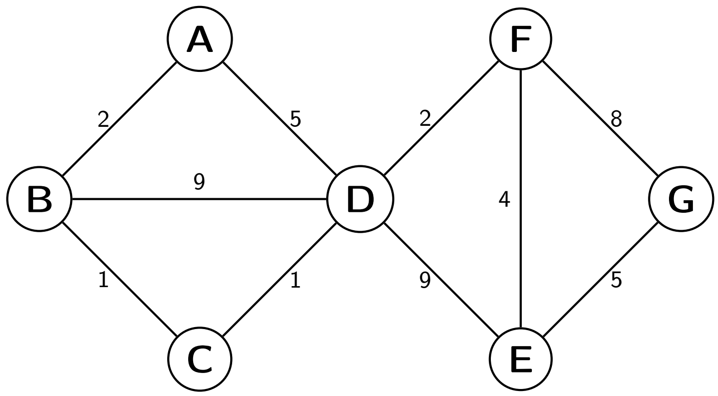
1. (2.5 points) Assume both players act optimally. Provide the minimax values of nodes A to G.
2. (5 points) Assume the search visits nodes from left to right (i.e. the left-most unvisited child is chosen first). Through alpha-beta pruning, show your alpha and beta values at each node after the last time it is visited. Which nodes are pruned from the game tree above? If no nodes can be pruned, why not.
3. (2.5 points) In the context of this problem, explain, in plain English, would you want to use alpha-beta pruning over naive minimax. How about larger trees?

Now suppose we are in a non-deterministic environment with the following rules. At the beginning of each turn, a player first flips a fair coin and if it turns up head, the player will act optimally. On the other hand, if the coin turns up tail, the player will pick one of the three available actions uniformly at random.

4. (5 points) Given the non-deterministic scenario above, redraw the game tree by introducing a chance node C as the only successor of each non-terminal node X, while the original successors of X are now successors of C. Label the probability on each edge from C to its successors. These probabilities should sum up to 1 for each C.
5. (5 points) Provide the expectiminimax values of nodes A to G. Round your answers up to the nearest hundredth.

Question 3: Heuristic [10 points]

Consider the undirected graph below. All edges are bidirectional and their costs are labeled as shown on the graph. A is the start node and G is the goal node.



Suppose we have two heuristic functions as follows:

Node	h_1	h_2
A	14	12.5
B	14	12
C	10	11
D	5	10
E	6	5
F	2	5.5
G	0	0

1. (2.5 points) Is h_1 admissible? Is h_2 admissible? Explain your reasons.
2. (2.5 points) Provide all possible paths that could be returned by A* search using h_1 versus h_2. Do they return different paths? Briefly explain.

Suppose you have an incomplete heuristic function h_3 below and the value for $h_3(D)$ is missing.

Node	A	B	C	D	E	F	G
h_3	11	4	10	?	5	5.5	0

3. (2.5 points) What real interval of $h_3(D)$ would result in h_3 being admissible?
4. (2.5 points) What real interval of $h_3(D)$ would result in h_3 being admissible, and A* search to expand nodes A, B and C in a sequential order?