A Mathematical model for Optimal Russian Water Strategy

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2/4/2013

Abstract

We have created a mathematical model based on a computer simulation of the effects of certain water strategies. We have classified the potential strategies, their costs, and their impact on the Russian people. We have included the dynamic, nonlinear relationships between these strategies and standard measurable indicators of national success. We have implemented a Monte Carlo scheme to find an optimal set of decisions for the Russian water department. We inserted real world data and found a particular strategy best suited for the well being of the Russian people. We formulated a position paper to explain our model, its scope, and its recommendations to Russian government leadership. Our model is superior as it is inherently computational, simple to use, and allows for easy and extensive appraisal, modification, and refinement.
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1 Introduction

1.1 Analysis of the problem

The problem asks us to come with effective, feasible and cost-effective strategy for 2013 that will fulfill fresh water need for Russia in the year 2025. The mathematical model we develop should address storage, movement, treatment and conservation of water while keeping an eye on economic, physical, and environmental implications. At the end we are asked to present a non-technical summary addressing how our approach will be appropriate to the Russian authorities based on their requirement for cost-effectiveness and feasibility.

1.2 Outline of our approach

We propose to use a computer simulation to model the effects of Russia’s water strategy. This simulation takes specific actions as input, calculates and updates critical variables, and outputs a generalized score. This generalized score is a weighted average of several independent factors important for evaluating the success of a water strategy. The mathematics of this model and of this problem are located in the nonlinear dependence of the variables on the inputs. For example a specific action might be investing in a water treatment plant. This action will decrease the amount of available money and also decrease the average medical costs of the Russian populace. Our generalized score will included the final average medical costs as one of its weighted independent factors. We optimize this model using a Monte Carlo approach. We run many trials with random decisions. If the final score of a given trial is the highest so far we record its set of decisions. After performing many trials we hope to find a set of decisions that best addresses Russia’s water needs.

1.3 Our Assumptions

We begin by diving Russia into four different regions- St. Petersburg, Moscow, Sakha and Dagestan region. These regions represent Russia as whole in terms of population dynamics and industrial spread. Under this assumptions, if our model predicts optimum water supply investment for any of above mentioned regions or any combination of these regions, it can be readily expanded by adding more regions of Russia.

Further more, the sources of water for St. Petersburg and Dagestan are oceans and thus these regions have options of desalination plants. However, Moscow and Sakha regions being landlocked, the sources of water in these regions are rivers and lakes.

Under our simulation, a user is asked to select a region followed by input which asks him/her to input numbers for various variables- number of pumping stations, water treatment plants, pipelines for transportation and storage facilities. For each of these input, there is certain weight, as mentioned in the outline. At the end of the run, total weight of the run is computed based on the input from the user. This numerical value for the weight is used to compare any two or more runs to determine the successful run. The successful run is the one with the highest total score.
<table>
<thead>
<tr>
<th>Quantity</th>
<th>Units</th>
<th>Value</th>
<th>Source(Reference)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Population</td>
<td>millions of people</td>
<td>143</td>
<td>7</td>
</tr>
<tr>
<td>Initial Growth rate</td>
<td>Percent people per half year</td>
<td>0.05 percent</td>
<td>7</td>
</tr>
<tr>
<td>Carrying Capacity</td>
<td>millions of people</td>
<td>0.0015</td>
<td>An educated guess based on the population of China and the size of Russia</td>
</tr>
<tr>
<td>Water Consumption per capita</td>
<td>gallons per half year</td>
<td>0.011</td>
<td>6</td>
</tr>
<tr>
<td>Total Water Stored</td>
<td>millions of gallons per half year</td>
<td>30</td>
<td>1</td>
</tr>
<tr>
<td>Total Water Supply</td>
<td>millions of gallons per half year</td>
<td>23700</td>
<td>6</td>
</tr>
<tr>
<td>Cost per source plant</td>
<td>millions of dollars</td>
<td>8.7</td>
<td>2</td>
</tr>
<tr>
<td>Cost per capacity</td>
<td>millions of dollars</td>
<td>821</td>
<td>1</td>
</tr>
<tr>
<td>Capacity per cost</td>
<td>millions of gallons</td>
<td>4106</td>
<td>1</td>
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<tr>
<td>Water supply per plant</td>
<td>millions of gallons per half year</td>
<td>18000</td>
<td>2</td>
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<tr>
<td>Price per gallon</td>
<td>millions of dollars</td>
<td>0.08</td>
<td>3</td>
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<tr>
<td>Cost per purification</td>
<td>millions of dollars</td>
<td>33.3</td>
<td>4</td>
</tr>
<tr>
<td>Life expectancy without purification</td>
<td>years</td>
<td>75</td>
<td>7</td>
</tr>
<tr>
<td>Life expectancy with purification</td>
<td>years</td>
<td>70</td>
<td>Our estimate</td>
</tr>
</tbody>
</table>

### 2 The Model

#### 2.1 Rules of the Game

Our game consists of twenty-four time segments. Each year is divided into two time segments, allowing for the season dependence of water usage. In each time segment there is a series of three events. First the player chooses potential actions from a list of possibilities. Each possibility has a cost and defined impact on a set of variables. The possibilities are broken down into categories, then into subcategories corresponding to the region, the type, and the quantity. After the user has input their choices the simulation updates internal quantities. For example, this simulation calculates the money received for supplying citizens with water. The water department's budget is updated. Thirdly, the vector of external factors is reported back to the user. This ends the time segment. After twenty-four time segments each factor is calculated. The four factors we used are: money available, percentage of demand served, average life expectancy, and population. The final weight was calculated so that each component had a roughly equal contribution to the total score. The weight for money available is 1/2000. The weight for percentage of demand served is 5. The weight for average life expectancy is 1. The weight for population is 1/1000. The final score is the sum of all of these. A graphical representation is as follows:
The demand for water in our model is linearly dependent on the population with a given fixed rate. Further plans would allow for this rate to be variable depending on conservation efforts.

We calculate the water usage as follows. If there is less supply than demand we supply the rest with whatever water has been stored, exhausting the storage if possible. If there is more supply than demand we add the rest to the storage, up to the storage capacity. We calculated percentage served as the total served including storage divided by the demand. We increase the money by a constant rate times the amount served.

We calculated the change in population using a logarithmic growth model. We specified the initial population, growth rate, and carrying capacity from numbers found in references and guided estimates. We assume having adding a purification plant will increase the population growth rate due to its positive effects on health.
2.2 Optimization

To find an optimal solution we run the simulation with random computer generated inputs. We keep track of the best score and the decisions associated with the best score. After each trial we compare its score with the best score. If its score exceeds the best score we replace the past best score and past best decisions with the current score and decisions. We run for many trials and then note the final best score and the final best decisions. We use the set of decisions as our recommendation to the water committee. For more justification we randomly generated decisions and then chose final sets of decisions with a specific property, say a third of decisions were to add a new source. We then find an average score for sets that have this property. We do this for a representative selection of possibilities. We then can compare different prospective strategies. For instance, we find that the average score for adding sources a third of the time is 56, but the average score for adding sources three fourths of the time is 65. We then can infer that adding sources is a successful strategy. By choosing an appropriate sample set of priorities one should be able to understand the system more fully.

3 Results

These set of graphs are meant illustrate the effect a particular strategy might have on the score. Each data point represents the score obtained by the particular strategy. Strategy 0 is do nothing. Strategy 1 is build one source plant. Strategy 2 is to build 1 storage plant. Strategy 3 is to build a sanitation plant. For instance in graph 0 each data point has chosen 'do nothing' 12 out of 24 rounds. Then, we calculate the mean and display it as the horizontal bar. Each graph also includes standard deviations from the means as the error bars for each point. We see that the solutions with larger error bars are less influential in determining the score.
 std dev = 1.756942
std dev = 1.910886
This graph shows the relative merits of five different strategies. The first four are pure strategies obtained by doing the same action each time. The last is a mixed strategy chosen using our Monte Carlo method. We note that the mixed strategy is the second best indicating some level of fitness. The highest is to always add a purification plant. We note that this arises because the parameters we inputed suggested that the Russian demand is currently very highly met. Any new source or storage facilities would only marginally increase the factor of percentage supplied. In contrast the level of purification is somewhat lower and additional methods of purification have an ability to increase the lifetime expectancy in a meaningful way. Do note the important inclusion of the do nothing strategy. This is a good benchmark for determining the value of any action. Its higher level of success than the source or storage option indicates, probably accurately, that the Russian government should note waste money on adding storage or sources.
4 Summary

4.1 Strengths

1. Our model is simple. The mathematical methods used can be understood by an average adult with little mathematical knowledge. Politics and politicians are suspicious of overly and overtly complicated models. Our model includes the complications and nonlinearities into a framework easily understood by the lay person.

2. Our model is intuitive. It is not hidden by variables, equations, or mathematical methods. There is a one to one correspondence between actions in real life and their effects in our model.

3. Our model is highly flexible. The same process and computation is involved regardless of the values of the assumed quantities. It is easily updatable.

4. Our model easily allows for variable rates of sophistication. More detail can be implemented in a straightforward fashion without the reorganization of central mechanics.
5. Our model can easily generalize to other situations. We can use the same framework to 
analyze the best electricity strategy or the impact of new types of desalination plants. 
We can combine the existing framework and some of the above proposals into a single 
coherent model allowing for holistic approach.

6. Our model is inherently computational. This allows for easy access to numerical results. 
There is no loss in translation or effort to interpret our results.

4.2 Weaknesses

1. Our model is highly non analytic. It would be very hard to find the absolute best 
answer. The calculations cannot be made by a human. It is not proposed in canonical 
mathematical language.

2. Its current state is simplistic and without detail. The practical problems, for example 
of desalination or distribution structure, are reduced to costs and benefits. It treats 
these problems superficially.

3. The optimization algorithm is not smart. Inputing random choices is an inefficient way 
of searching the entire set of possibilities. Our given solutions offers little justification 
for the recommended choices.

4. Our model is weakly specific to Russia. What differentiates Russia from other countries 
in our model is only the set of parameters input into the model.

4.3 How should you use it?

Our model is intended to be used a tool for the Russian water committee. We have supplied 
our estimates for the costs of given plants or lines but these values are only educated guesses 
based on our available information. We assume that the Russian government will have 
much more accurate values. Our model allows them to simply insert whatever values they 
propose and calculates an optimal solution. Similarly we provided educated guesses for 
the relative values of the independent factors. The Russian government may value these 
factors differently than us or differently at future times. Our model is easily updated to 
accommodate there varying interests. In sum, we have created a model to be used as a tool 
for the Russian government. Our valuations merely serve as a provisional estimate and an 
example of use.

4.4 Places for Further Development

To further develop this model we would create a smarter optimization procedure. Instead of 
random choices our method would make choices based on the success of previous results. An 
intelligence based on a neural network or simulated annealing algorithm. This would help 
solve weakness number one. In a similar tack, our optimization algorithm might keep certain 
decisions constant and change one dependent variable. In this way we could see the effects 
of a particular decision. This would help solve weakness number three. We could then plot
the generalized score as function of the frequency of the particular decision that is varied. We could then combine the graphs for each possible decision to try to find the multivariate equation for the general score in terms of the possible decisions. In this way we could reverse engineer an equation attainable to mathematical analysis. This would help solve weakness number one. Citizen science and civil involvement using mobile apps or similar technology has been an emergent feature of the twenty-first century. We envision converting our model to a game similar to the popular Foldit. We can then allow the citizens to more active in the government, utilize a highly intelligent resource, obtain a human understanding of the justification of our proposal, and refine our model to more in tune with actuality.

4.5 Testing

To test our model we have multiple possibilities. One possibility is to load in starting data for an earlier time, say 2002, load in the actual choices of the Russian government, and then compare the results of our model to the known end quantities in 2013. We can then use the discrepancy between our model and actuality as and error bar for future events. By running the model for many different starting points and including situations of unforeseen impact, say the dissolving of the USSR, we can evaluate the robustness of our model. Since our model is highly flexible we can slightly change the input variables and compare the optimal strategies for the different situations.

5 Position Paper

In policy making, the Russian government must consider many effects of any given action. This can be a daunting and imprecise task. We have built a mathematical model to assist in this task. Our model allows for the evaluation of a particular policy in terms of standard measurable quantities. We can then assign importance to these qualities to find a "rating" of the particular policy. Following this approach we can generate and evaluate potential policies much faster and more precisely than humanly possible. In this way we can apply the power of computation to governance and public policy. In light of the recent economic collapse precipitated in part because of models of this sort, but specifically by their obscurity and inaccessibility, we have created meaningful, easily understandable ways to investigate the behavior of this model. We have elucidated the complex mathematical dependence in terms of simple casual relationships understandable to the average person. Moreover our model is highly modular and its assumptions are easily accessed. As a government official, we offer you the tools investigate the effects of your proposals with minimal intellectual work. We do not require you to do any investigation if you so choose. We have input our own parameter based on educated analysis, and supplied reasonable preferences. We have run the model ourselves and come up with a best proposal for Russia’s water supply. We propose that the Russian government implement one purification plant per year. We modeled the situation and found that this strategy maximizes the benefit to the Russian populace. We found the reason for this is that given Russia’s current situation the demand for water is highly met and any new storage or sources would give marginal gains to those served or proposed income. Whereas the Russian people’s sanitation levels are relatively lower and an increase...
in purification plants would significantly increase lifetime expectancies. Moreover this benefit to lifetime expectancies would be enough to offset the cost to build such plants. In summary we have built a model to simulate the effects of Russia’s water strategy, input real world parameters describing Russia’s current situation, and have found that the best strategy is to build water purification plants. This is our recommendation to your department.

References


