6.1 Large-Sample Tests for a Population Mean

In chapter 4, we found that for a population with mean $\mu$ and standard deviation $\sigma$, $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$ in two important cases:

- for $\underline{n}$ for (almost) any population (then “$\sim$” is $\underline{\text{ }}/\underline{\text{}}$)
- for $\underline{n}$ only for a normal population (then “$\sim$” is $\underline{\text{ }}/\underline{\text{}}$)

In chapter 5, we used “$\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$” to make confidence intervals for $\mu$. In §6.1 we use this fact to test hypotheses about $\mu$.

Definitions and a First Example

A hypothesis test checks whether a parameter value $\underline{\text{ }}/\underline{\text{}}$ with a sample. It considers

- $H_0$, the null hypothesis, which asserts that “any effect indicated by the sample is merely due to $\underline{\text{ }}/\underline{\text{}}$, and is $\underline{\text{ }}/\underline{\text{}}$ an effect in the population”; and
- $H_1$, the alternative hypothesis, which $\underline{\text{ }}/\underline{\text{}}$ $H_0$ and says “the effect in the sample is $\underline{\text{ }}/\underline{\text{}}$ in the population”

$H_0$ is presumed until evidence makes $\underline{\text{ }}/\underline{\text{}}$ it unreasonable.
e.g. An environmental standard specifies that the mean dissolved oxygen content \( X \) in a stream should be greater than 5 mg/L. One-liter water samples from 45 random stream locations have \( \bar{x} = 4.62 \text{ mg/L} \), with \( s = 0.92 \text{ mg/L} \). Is this strong evidence that the stream has a (population) mean \( O_2 \) content \( \mu \) less than 5 mg/L, suggesting that a nearby polluter should be penalized?

We test

- \( H_0 : \mu = 5 \) (this could be any \( \mu \geq 5 \), but \( \mu = 5 \) most favors the innocence of the factory) against

- \( H_1 : \mu < 5 \)

The *null distribution* of a statistic is its distribution under \_______________________.\

e.g. Find (and draw) the null distribution of \( \bar{X} \), the sample mean oxygen content.

A *test statistic* compares the \______________________ value of a parameter to an \________________ of the parameter from sample data. A large value of the test statistic indicates an estimate \________________ from the parameter, which is evidence \________________ \( H_0 \).

e.g. Standardize \( \bar{X} \) to get the test statistic \( Z = \frac{\bar{X} - \mu}{\sigma_x} = \_______________________ \), which tells how far \( \bar{X} \) is from \( \mu \), in standard deviations. Since \( \bar{x} = 4.62 \text{ mg/L} \), \( z = \_______________________ \).

A test’s *P-value* is the probability, under \( H_0 \), of a result \_______________________ as the value of the test statistic (draw). The smaller the P-value, the \________________ the evidence against \( H_0 \).

e.g. Find the P-value of the oxygen test. Should the polluter be penalized?
A Two-Sided Test

The preceding test is *one-sided* because a test statistic value in only ______________ of its distribution is evidence against $H_0$. Next is a *two-sided* test, in which a value in ______________ is evidence against $H_0$.

e.g. (p. 220 #6) A stainless steel powder is supposed to have a mean particle diameter of $\mu = 15$ $\mu m$. A random sample of 87 particles had a mean diameter of 15.2 $\mu m$, with standard deviation 1.8 $\mu m$. Test whether this is evidence that the powder doesn’t meet its specification.

We test $H_0 : \text{_______________} \text{against} \ H_1 : \text{_______________}$

The null distribution of $\bar{X}$ is \ldots

Find the $P$-value. Does the powder meet its specification?

Summary

Suppose $X_1, \cdots, X_n$ is a large ($n > 30$) random sample from a population with mean $\mu$ and standard deviation $\sigma$. To test that $\mu$ has a specified value, $\mu_0$,

1. State null and alternative hypotheses, $H_0 : \mu = \mu_0$ and $H_1$ (below)

2. Check assumptions

3. Find the $z$-score, $z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$ (if $\sigma$ is unknown, use $\sigma \approx s$)

4. Find the $P$-value, which depends on $H_1$:
   $H_1 : \mu > \mu_0 \implies P$-value $= P(Z > z)$, the area right of $z$
   $H_1 : \mu < \mu_0 \implies P$-value $= P(Z < z)$, the area left of $z$
   $H_1 : \mu \neq \mu_0 \implies P$-value $= P(|Z| > |z|)$, the sum of areas left of $-|z|$ and right of $|z|$

5. Draw a conclusion.
Extra Example

A dynamic cone penetrometer (DCP) measures material resistance to penetration (mm/blow) as a cone is driven into pavement or subgrade. Suppose an airport runway requires a true average DCP value below 30. Its pavement will not be used unless there is strong evidence that the specification has been met. Measurements at a random sample of 52 locations in the runway had mean $\bar{x} = 28.76$ and standard deviation $s = 12.26$. Should the runway be used?