6.2 Drawing Conclusions from the Results of Hypothesis Tests

Recall from §6.1 that a test’s \( P \)-value is the probability, under \( H_0 \), of a result

- The smaller the \( P \)-value, the ________ the evidence against \( H_0 \).
- A large \( P \)-value doesn’t ________ \( H_0 \). It just leaves it ________.
- A rule of thumb (___________ scientific justification) suggests rejecting \( H_0 \) for ________.
  - If \( H_0 \) is well-established, or if the consequences of rejecting it are high (e.g. long prison sentence), reject \( H_0 \) only for a __________ \( P \)-value.
  - A preliminary study, done to decide whether a line of research is worth pursuing, might reject \( H_0 \) with a __________ \( P \)-value.

The conclusion of a hypothesis test is ________ or ________ (not ________).

Statistical Significance

A test result is statistically significant (and \( H_0 \) is ________) at the (100%) \( \alpha \) level if ________.

e.g. When testing mean dissolved oxygen content, \( H_0 : \mu = 5 \) vs. \( H_1 : \mu < 5 \), we found \( P = .0028 = .28\% \). At which levels is this result significant? 5%: ________, 1%: ________, .1%: ________

Don’t just report only “\( H_0 \) is rejected at the 5% level”: also report ________.

Caution: A result is statistically significant if it’s unlikely to be due to random sampling variation, which has nothing to do with __________. Large random samples have small variation (\( \sigma_X = \sigma/\sqrt{n} \)), so effects small enough to be ________ can be statistically significant.

The \( P \)-value Is Not the Probability That \( H_0 \) Is True

It’s the probability, __________, of a result as extreme as the value of the test statistic.
The Relationship Between Hypothesis Tests and Confidence Intervals

These two statements are equivalent:

- A level-$\alpha$ test of $H_0: \mu = \mu_0$ vs. $H_1: \mu \neq \mu_0$ rejects $H_0$ (because it's that $\mu = \mu_0$, in light of the sample mean $\bar{x}$).

- $\mu_0$ falls a $1-\alpha$ confidence interval for $\mu$ (a range of values for $\mu$, in light of the sample mean $\bar{x}$).

For this $\bar{x}$, $H_0$ is and $\mu_0$ is the confidence interval for $\mu$:

\[
\bar{X} \sim N(\mu_0, \sigma^2/n)
\]

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\]

e.g. The $P$-value for a two-sided test of $H_0: \mu = 10$ vs. $H_1: \mu \neq 10$ is 0.06.

a. Does the 95% confidence interval for $\mu$ include 10? Why?

b. Does the 90% confidence interval for $\mu$ include 10? Why?
A radon abatement device (e.g. a vent pipe system and a fan to pull radioactive radon-laden air from beneath the house) is recommended in any home where the mean radon concentration is 4.0 picocuries per liter (pCi/L) or more. The mean of 75 measurements in a home is 3.72 pCi/L, with standard deviation 1.93 pCi/L.

a. The inspector who did the test says, “since the mean measurement is less than 4.0, abatement is unnecessary”. Why is this reasoning incorrect?

b. Radon abatement is recommended whenever it’s plausible that the mean concentration is 4 or more. State $H_0$ and $H_1$ for determining whether abatement is appropriate.

c. Find the $P$-value. Do you recommend abatement?

Choose $H_1$ to Answer the Right Question

e.g. (Similar to p. 227 #9a) Suppose an epoxy will be used to bond wood if its mean shear stress is greater than 10 MPa. Which $H_0$ and $H_1$ regarding the population mean $\mu$ are most appropriate?

- $H_0 : \mu = 10, H_1 : \mu < 10$
  - Rejecting $H_0 \implies$
  - Retaining $H_0 \implies$

- $H_0 : \mu = 10, H_1 : \mu \neq 10$
  - Rejecting $H_0 \implies$
  - Retaining $H_0 \implies$

- $H_0 : \mu = 10, H_1 : \mu > 10$
  - Rejecting $H_0 \implies$
  - Retaining $H_0 \implies$
e.g. (p. 226 #2) True or false:

a. If we reject $H_0$, then we conclude that $H_0$ is false.
b. If we do not reject $H_0$, then we conclude that $H_0$ is true.
c. If we reject $H_0$, then we conclude that $H_1$ is true.
d. If we do not reject $H_0$, then we conclude that $H_1$ is false.

<table>
<thead>
<tr>
<th><strong>Strict logical answer</strong></th>
<th>Practical decision</th>
<th>Book’s answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td></td>
<td>True</td>
</tr>
<tr>
<td>b.</td>
<td></td>
<td>False: $H_0$ is plausible.</td>
</tr>
<tr>
<td>c.</td>
<td></td>
<td>True</td>
</tr>
<tr>
<td>d.</td>
<td></td>
<td>False: $H_0$ is plausible, so $H_1$ is plausible too.</td>
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