CS515 Spring 08

Prof. Ron

Assignment #7

Due 29 April 2008

Question # 1.

You experiment in this question with the error bound in spline approximation. To this end, take your interval to be $[-2, 2]$, and choose several really smooth functions on that interval (take $t \mapsto \cos t$, $t \mapsto 1/(3+t)$ and another function of your choice.) You will be using cubic spline approximation, with three different selections of the linear functionals:

- $\lambda_j f = f(t_j)$ (i.e., the coefficient of the B_j B-spline is evaluation at the knot t_j). The resulting approximation scheme is exact on constants, but not on linear polynomials.
- $\lambda_j f = f(t_j^*)$, where t_j^* is the dual knot (i.e., for cubic spline this is $(t_{j+1} + t_{j+2} + t_{j+3})/3$.) The resulting approximation scheme is exact on linear polynomials, but not for quadratic polynomials.
- λ_j is the de-Boor-Fix dual functional (corresponding to cubic splines). Take the point τ_j that appears in the definition of λ_j to be t_{j+2}. The resulting approximation scheme is exact on cubic polynomials.

Now, choose 41 equidistant knots on $[-2, 2]$, and then 81 equidistant knots on that interval, and apply for each case each of the three approximation schemes to each of your three smooth functions.

Turn in: (1) the `Matlab` code that you used in order to evaluate the various approximations to your various functions. (2) Plots of the error function for each approximation. (3) The theoretical error bounds in each case, based on the error analysis we did in class. (4) Comments and conclusions of any type.

Comments:

You can use here any routine from the Spline Toolbox for evaluating splines. For example, we can use the B-spline evaluation routine from the last assignment.

Also, you will need, for the de Boor–Fix dual functionals to calculate the derivatives of the function you approximate. You may calculate those manually and input them in this way to your scheme.

Finally, note that you should not evaluate your error too close to the end points 2 and -2 since you are missing a few requisite B-splines near the end points. You will need to find out yourself the correct interval where your theoretical error bounds should be valid. Do the numerical error evalutions on those intervals only.