CS514 Spring 00
Prof. Ron

Assignment #7

Due 03 May 2000

Question # 1.
Verify directly that, for any polynomials q, p of order k, the number

$$\sum_{\nu=1}^{k}(-D)^{\nu-1}q(\tau)D^{k-\nu}p(\tau)$$

is independent of τ. (Hint: differentiate wrto τ.) Why is this relevant to the course material?

Question # 2.
Write a `Matlab` function `sptayl(a,t,x)` that inputs a coefficient sequence `a`, of length k, and, correspondingly, a knot sequence `t` of length $2k$, and returns the vector $[p_k(x); Dp_k(x); \ldots; D^{k-1}p_k(x)]$, with p_k the polynomial that agrees with the spline $f := \sum_{j=1}^{k} \mathtt{a}(j)B_{j,k}$ on the interval $(t_k \,..\, t_{k+1})$.

Start off with `Matlab` code for the calculation of $p_k(x)$ via the recurrence (feel free to make use of the code in the lecture notes for 17apr). Enlarge this code by differentiating it $k-1$ times appropriately, as indicated in the lecture notes for 17apr, and so getting the needed derivatives along with the value.

Use your `sptayl` to obtain the vector $[f(2); Df(2); D^2f(2); D^3f(2)]$ for the cubic B-spline $f = B(\cdot|0,1,2,3,4)$. (You will have to augment this B-spline's knot sequence appropriately.) You already know that `sp = spmak(0:4,1)` provides this f, and that `fnval(sp,x)` provides its value at `x`. If you add to this that `dsp = fnder(sp)` provides the first derivative of the spline in `sp`, you are all set to check out your results.

Hand in a listing of your m-file `sptayl` along with a (well-commented) diary that records your use of `sptayl` and the check of your results.

Question # 3.
(a) Use `spmak`, `fn2fm`, and `fnbrk` (as indicated in the lecture notes for 24apr) to obtain the matrix C with the following property: *For every cubic spline* $f = \sum_j a_j B_{j,4,\mathbb{Z}}$ *with knot sequence* $\mathbb{Z}$, *and for every* $x \in [0\,..\,1)$,

$$f(x) = (1/6)[x^3, x^2, x, 1] * C * [a_{-3}; a_{-2}; a_{-1}; a_0].$$

(The $*$ is used here to indicate ordinary matrix multiplication, and the division by 6 is put here to make certain that all the entries of C are integers.) Justify all the steps in your derivation.

(b) Use the matrix C that you obtained in (a) to compute the value of $f := \sum_j (j+2)B_{j,4,\mathbb{Z}}$ at 5.75.

Question # 4.
In an earlier assignment, you considered the *cardinal* cubic B-spline $B_4 := B_1 * B_1 * B_1 * B_1$, obtained by *convolving* $B_1 := \chi_{[0..1)}$ with itself. (Be alert to the fact that, in this Question, the star, $*$, indicates convolution, and that B_j is *not* the jth B-spline for some unspecified knot sequence.) Try to prove that this cardinal cubic B-spline is 0th cubic B-spline for the knot sequence $\mathbb{Z}$, i.e., equals the (recursively defined) B-spline $B_{0,4,\mathbb{Z}} = B(\cdot|0,1,2,3,4)$. (Hint: if $g : x \mapsto (B_1 * f)(x)$, what is its first derivative, Dg, in terms of f? Is there somewhere a formula for the first derivative of a spline that might be useful here in an inductive argument?)