quick review of spline part of CS514, Spring'00

Focus was on recovery (exact or partial) of a function g in some linear space X of functions from some data $\Lambda' g$ about it, with $\Lambda' : g \mapsto (\lambda_i g : i \in I)$ a linear map into the coordinate space $\mathbb{F}^I$. (The discussion involved the terms: linear, 1-1, onto, invertible, range, kernel, linear (sub)space, basis, coordinate space, data map, analysis operator, recovery map, synthesis operator, and the totally nonstandard term 'column map'.)

If the recovered function is to be in the linear subspace F with basis W, then $f = Pg = W(\Lambda' W)^{-1}\Lambda' g$ is the unique element of F that matches the given information (i.e., $\Lambda' f = \Lambda' g$), provided the Gramian matrix $\Lambda' W = (\lambda_i w_j)$ is invertible.

In that case, also $V := W(\Lambda' W)^{-1}$ is a basis for F, and $\Lambda' f$ are the coordinates of f wrto that basis. Hence our recovery amounts to nothing more than a change of basis. But such a change of basis is useful if the information about f (or g) we are after is more readily obtainable from $(\Lambda' W)^{-1}\Lambda' f$ than from $\Lambda' f$.

This raises the basic question of which information about $f \in F$ is readily available from its coordinates wrto a given basis for F.

We explored this question for $F = \Pi_{<n}$, comparing Lagrange basis and power basis.

We explored this question in very great detail for F a space of more or less smooth piecewise polynomial functions, with special focus on its B-spline basis (but also some mention of the ppform, which employs a basis for the space $\Pi_{<k,\boldsymbol{\xi}}$ of all piecewise polynomials of order k with break sequence $\boldsymbol{\xi}$).

Specifically, we discussed altogether 12 (of course!) B-spline properties, including the recurrence relation used in their definition, their support, number of nontrivial polynomial pieces, conventions and formula concerning their differentiation, Marsden's Identity, the recipe for constructing the B-spline basis for $\Pi_{k,\boldsymbol{\xi},\boldsymbol{\nu}}$, the Schoenberg-Whitney theorem, the convex hull property, the dual functionals, local and nonnegative partition of unity, knot insertion (refinability), control points and polygons, and variation diminution.

While there was some discussion concerning the construction of a spline, especially a spline curve, by explicit specification of its control points, we mainly discussed construction of a spline by interpolation to some data, including the proper choice of knot sequence for that, and finished (much too early :-) with a discussion of least-squares spline approximation, stressing again the underlying maps.

All details are available in the online lecture notes.

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