Final Exam, CS515, Fall03: list of formulæ.

Definition of a B-spline (of order $k \geq 2$ and knots $(t_i, \ldots, t_{i+k})$):

$$B_{i,k} = \omega_{i,k} B_{i,k-1} + (1 - \omega_{i+1,k}) B_{i+1,k-1}, \quad \omega_{i,k}(t) := \frac{t - t_i}{t_{i+k-1} - t_i}.$$

Recursive evaluation of B-spline coefficients: If $f = \sum_j a_j B_{j,k}$, then $f = \sum_j a_j^{[2]} B_{j,k-1}$, with

$$a_j^{[2]} = a_j \omega_{j,k} + a_{j-1}(1 - \omega_{j,k}).$$

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de Boor - Fix dual functionals:

$$\lambda_{jk} f := \sum_{\nu=1}^{k} \frac{(-D)^{\nu-1} \psi_{jk}(\tau)}{(k-1)!} D^{k-\nu} f(\tau),$$

with τ any fixed point in the support of the B-spline B_{jk}. Here,

$$\psi_{jk}(\tau) := (t_{j+1} - \tau) \cdots (t_{j+k-1} - \tau).$$

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knot insertion: if $f = \sum_j a_j B_{j,k,\underline{t}}$, and $\widehat{\underline{t}}$ is obtained by adding a knot x to $\underline{t}$, then

$$f = \sum_j \widehat{a_j} B_{jk,\widehat{\underline{t}}},$$

with

$$\widehat{a_j} := \widehat{\omega}_{jk}(x) a_j + (1 - \widehat{\omega_{jk}}(x)) a_{j-1},$$

where

$$\widehat{\omega_{jk}}(x) := \max\{0, \min\{1, \omega_{jk}(x)\}\}.$$