UNIVERSITY OF WISCONSIN - MADISON

Computer Sciences Department

CS514, Spring 02

Prof. Ron

Midterm Exam: sample questions

Date: March 11, 7:00-9:00 p.m.

Closed books. Calculators are allowed.

The sample is merely aiming at illustrating the kind of questions you might expect on this exam. Of course, the sample is not comprehensive, only illustrative.

(1) (points).
Define the notion of a CQF mask. Check whether the function

$$H(\omega) = \cos^2(\omega/2)$$

is a CQF.

(2) (points).
Check whether the following assignment:

$$\lambda : f \mapsto \int_0^1 t^3 f'(t)\, dt$$

is a linear functional.

(3) (points).
Let $\mathcal{D}_a$ be the normalized dilation operator:

$$\mathcal{D}_a f(t) := \sqrt{a} f(at).$$

Now, assume that f is 2π-periodic. Is $\mathcal{D}_2 f$ necessarily a 2π-periodic function, too? Explain, and discuss at list one example.

(4) (points).
Find the Fourier transform of the function

$$f(t) := \begin{cases} 1, & 0 \le t \le 10, \\ 0, & \text{otherwise.} \end{cases}$$

(5) (points).
You are given the function:

$$f(t) = \frac{1}{1+t^2}.$$

Without computing $\widehat{f}$, what can you say about (i) the smoothness, (ii) decay, of $\widehat{f}$? Explain.

(6) (points).
Is the function f whose Fourier transform is given by

$$\widehat{f}(\omega) = \frac{\sin(\omega/2)}{\omega/2}$$

refinable? If your answer is 'yes' find its refinement mask. If your answer is 'no', explain why.

(7) (points).
You are given the following `matlab` code:.... Explain what the code does, and write down its output.

(8) (points).
You are given the following `Matlab` code. Note that the code contains three loops. Rewrite the code, using as few loops as possible.

(9) (points).
Let $H_0(\omega)$ be a trigonometric polynomial that satisfies

$$|H_0(\omega)|^2 = \cos^6(\omega/2) + 3\cos^4(\omega/2)\sin^2(\omega/2).$$

Your friend Jeff found a wavelet mask H_1 such that the pair (H_0, H_1) satisfies the unitary extension principle. What can you say about H_1 (without finding explicitly H_0)? what can you say about the wavelet ψ whose mask is H_1?