

CS536 Lecture 11

Tuesday 24 February 2015

Reminders:

- Reading
- HW5 assigned, Due March 2.

Last class:

- Parsing
 - CYK

Today:

- Top-Down Parsing

Parsing: Approaches (Review)

Top-down (“Goal-Driven”):

Bottom-Up (“Data-Driven”):

CYK Algorithm:

- Decides whether a string can be generated by a grammar
- Bottom-Up
- Takes worst-case $O(n^3 |G|)$ time
- Only works on a specific grammar form

Chomsky Normal Form:

All rules must be in one of two forms:

$$\begin{aligned} X &\rightarrow \mathbf{t} \\ X &\rightarrow AB \end{aligned}$$

The empty string may only appear on the RHS of the start symbol.

Why CNF?

Transforming a grammar to CNF

Eliminate useless rules (reduce bloat):

$$\begin{aligned} S &\rightarrow X \mid Y \\ X &\rightarrow () \\ Y &\rightarrow (Y Y) \\ Z &\rightarrow (X) \end{aligned}$$

Eliminate epsilon-rules:

$$\begin{aligned} A &\rightarrow \epsilon \mid N \\ B &\rightarrow \text{ID } (A) \\ C &\rightarrow A \text{ SEMICOLON } A \end{aligned}$$

Eliminate unit productions:

$$\begin{aligned} A &\rightarrow N \\ B &\rightarrow \text{ID } (A) \end{aligned}$$

Fix RHS terminals:

$$N \rightarrow \text{ID } , N$$

Fix RHS nonterminals:

$$X \rightarrow ABC$$

Exercise: Convert the following grammar to CNF

$$\begin{aligned} F &\rightarrow \text{ID } (A) \\ A &\rightarrow \epsilon \mid N \\ N &\rightarrow \text{ID } \mid \text{ID } , N \end{aligned}$$

Parsing Algorithms

CYK is nice and works on all (CF) grammars ... but expensive!

We can do better for subsets of grammars. Linear time, even.

- LL (1)
- LALR (1)

Benefits of restricting grammars:

LL (1):

- Left to right (first L)
- Leftmost Derivation only (second L)
- Token lookahead of 1 (1)
- Top-down, “predictive” parser

LALR (1):

- Special lookahead procedure (LA)
- Left to right (second L)
- Rightmost Derivation (R)
- Token lookahead of 1 (1)
- Bottom-Up parsing

LALR(1) is strictly more powerful and also more difficult to understand.

Tradeoff: simplicity vs. power.

Top-Down Parsers

Start at the start nonterminal.

“Predict” what productions to use to reach terminal string.

General diagram:

Algorithm

```
stack.push(eof)  
stack.push(Start non-term)  
current_token = scanner.getToken()
```

Repeat

```
    if stack.top is a terminal y  
        match y with current_token  
        pop y from the stack  
        current_token = scanner.next_token()  
    if stack.top is a nonterminal X  
        get table[X,current_token]  
        pop X from the stack  
        push production's RHS (each symbol from R to L)
```

Until one of the following:

```
    stack is empty  
    stack.top is a terminal not matching current_token  
    stack.top is a non-term and parse table entry is empty
```

Example

Grammar: $S \rightarrow (S) \mid \{ S \} \mid \epsilon$

Parse Table:

	()	{	}	eof
S					

Example input: ({ }) EOF

Example bad input: ((} EOF

LL(1) grammars

Example grammar: $S \rightarrow (S) \mid \{ S \} \mid () \mid \epsilon$

Parse Table:

	()	{	}	eof
S					

How do we know then whether a grammar is LL(1)?

Also, how do we build the selector table?

Key: If each entry in the selector table has 1 production, the grammar is LL(1).

A grammar is *not* LL(1) if:

- It is left-recursive
- If it is not left-factored

Left-Recursion

Recall: A grammar is left-recursive (in X) if $X \Rightarrow^+ X\alpha$. It is *immediately* left-recursive if $X \rightarrow X\alpha$.

We can remove left-recursion without changing the language recognized.

Consider $A \rightarrow A\alpha \mid \beta$ (why must the second production exist?) where β doesn't begin with A .

Transform it to:

More generally:

Pictorially:

This introduces problems with associativity though ...

Left Factored Grammar

If a grammar is not left-factored, it cannot be LL(1).

Definition (of **not** being left-factored): A nonterminal has two or more productions whose RHS has a common prefix.

Example: $E \rightarrow (E) \mid ()$

This grammar is *not* left-factored.

Fixing this:

$$A \rightarrow \alpha\beta_1 \mid \alpha\beta_2$$

to

More generally:

Pictorially:

Combined Example

Grammar: $E \rightarrow (E) \mid E E \mid ()$

First, remove left-recursion:

Then, left-factoring

Another Example

Grammar:

$$\begin{aligned} \textit{Expr} &\rightarrow \textit{Expr} + \textit{Term} \\ &\mid \textit{Term} \end{aligned}$$
$$\begin{aligned} \textit{Term} &\rightarrow \textit{Term} * \textit{Factor} \\ &\mid \textit{Factor} \end{aligned}$$
$$\begin{aligned} \textit{Factor} &\rightarrow \textit{Exponent} ^ \textit{Factor} \\ &\mid \textit{Exponent} \end{aligned}$$
$$\begin{aligned} \textit{Exponent} &\rightarrow \mathbf{INTLIT} \\ &\mid (\textit{Expr}) \end{aligned}$$