Utilizing IMRT Intensity Map Complexity Measures in Segmentation Algorithms

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Radiation Teletherapy Goals

High Level Goals: Achieve prescribed radiation to tumor while minimizing dose to the surrounding organs

Some Additional Goals:
- Minimize time patient spends on the treatment couch.
- Minimize beam-on time (time radiation beam in “on”)
- Minimize treatment planning time

Multileaf Collimator (MLC)

IMRT: Beamlet Intensity Matrices

- Dose optimization yields beamlet intensity matrix at each selected beam angle
- Varying beamlet intensities:
  0 means no radiation,
  100 units is max dose
  (5 intensity level example)
- Using this matrix as input, a segmentation method computes a set of apertures needed to deliver the intensity map
Delivery of an Intensity Map via Shape Matrices

Intensity Map as Sum of Shapes

Given: Determine:

Intensity Matrix \( A \) = Sum of Shapes \( S_k \) times their weights \( \alpha_k \)

\[
A = \sum_{k=1}^{K} \alpha_k S_k
\]

\( \alpha_k > 0 \) is time the linear accelerator is opened to release uniform radiation

\( S_k \) is the binary shape matrix

Intensity Map as Sum of Shapes

A = \( \sum_{k}^{K} \alpha_k S_k \)

Multileaf Collimator Problem With Minimal Beam-on Time

What is “Optimal”?

• Given minimal beam-on time, minimize \#segments

• Given minimal \#segments, minimize beam-on time

• Minimize a weighted sum of beam-on time and \#segments
Difference Matrix

- The difference matrix for an IMap is the column-forward matrix of differences.

- Creating the difference matrix, $D$:
  - $A \in \mathbb{R}^{m \times n}$, $D \in \mathbb{R}^{m \times n + 1}$
  - Expand $A$ by adding a column of zeros to the left and to the right sides of $A$
    
    $A_{\cdot 0} = 0 \quad A_{\cdot n+1} = 0$

- Define $D$ row-wise by the differences:
  
  $D_{ij} = A_{ij} - A_{ij-1}$
  
  $i = 1..m$, $j = 1..n+1$
Some Basic Measures of Complexity

- numNonzeroIMap \( |A_{ij} > 0 | \)
- numNonzeroDiff \( |D_{ij} \neq 0 | \)
- numNonzeroDiffVert \( |V_{ij} \neq 0 | \)

Note: \( V \) is a “vertical” version of the difference matrix. i.e. \( V \) is the row-wise forward matrix of differences.

Complexity Which Might Affect #Segments: counts

- sumMaxUpDownSteps \( \sum_{i=1}^{n} \max\left( \text{count}_{j \in \mathbb{Z}}\{D_{ij} > 0\}, \text{count}_{j \in \mathbb{Z}}\{D_{ij} < 0\} \right) \)
- sumMaxUpDownStepsVert \( \sum_{j=1}^{n} \max\left( \text{count}_{i \in \mathbb{Z}}\{V_{ij} > 0\}, \text{count}_{i \in \mathbb{Z}}\{V_{ij} < 0\} \right) \)
- numValleys \( \text{count}\left(A_{ij} : A_{(i-1)j} \geq A_{ij} \geq A_{ij+1}\right) \)
- maxNumValleys \( \max\left( \text{count}\left(A_{ij} : A_{(i-1)j} \geq A_{ij} \geq A_{ij+1}\right) \right) \)
- maxMaxUpDownSteps \( \max_{i=1}^{m}\left( \max_{j=1}^{n}\{D_{ij} > 0\}, \max_{j=1}^{n}\{D_{ij} < 0\} \right) \)
- maxMaxUpDownStepsVert \( \max_{j=1}^{n}\left( \max_{i=1}^{m}\{V_{ij} > 0\}, \max_{i=1}^{m}\{V_{ij} < 0\} \right) \)

Lower Bound on #Segments

- IMap row
  - 6 steps up
  - 1 step down
  - lower bound of 6 segments

- total intensity increase of 80
  - lower bound of 80 units beam-on time

Complexity Which Might Affect Beam-on Time: sums

- sumStepsUpVert \( \sum_{i=1}^{m}\sum_{j=1}^{n}\{V_{ij}\} \)
- maxSumStepsUpVert \( \max_{j=1}^{n}\sum_{i=1}^{m}\{V_{ij}\} \)
- sumStepsUp \( \sum_{i=1}^{m}\sum_{j=1}^{n}\{D_{ij}\} \)
- maxSumStepsUp \( \max_{i=1}^{m}\sum_{j=1}^{n}\{D_{ij}\} \)
Lower Bound on Beam-on Time

- IMap row
- Profile of intensities
- Total intensity increase of 80
- 6 steps up
- 1 step down

Other Measures Considered: Poor Performers

- numDifferentStepSizes
- numDifferentValues
- Volume \( \sum_{i=1}^{m} \sum_{j=1}^{n} (A_{ij}) \)
- Perimeter \[ \begin{align*}
  \text{count}(A_{ij} : A_{ij} > 0 & \text{ and } A_{ij+1} = 0) + \\
  \text{count}(A_{ij} : A_{ij} > 0 & \text{ and } A_{ij+1} = 0) + \\
  \text{count}(A_{ij} : A_{ij} > 0 & \text{ and } A_{i+1,j} = 0) + \\
  \text{count}(A_{ij} : A_{ij} > 0 & \text{ and } A_{i+1,j} = 0)
\end{align*} \]

Random Intensity Map Generator: GenIMap

- Verified accuracy by:
  - Comparing average values of complexity measures to those of clinical cases
  - Comparing to a control set of IMaps generated using a uniform random distribution.
- Percent difference of complexity as compared to clinical cases:
  - GenIMap algorithm: **24% difference**
  - Uniform Random Distribution: **117% difference**

* 50 IMaps were generated for each algorithm

bDiff Heuristic

1. Choose weight, \( \alpha \).
   \[ \alpha = \min |D_{ij}| \]

2. Create candidate shapes.
   a) Create row-shapes.
      - \( D_{j+1} = \alpha \) and \( D_{j} = -\alpha \)
      - If a row has no such shapes, relax the rule to \( A_y \geq \alpha \) \( \forall i < j < r \)
   b) Combine row-shapes into larger shapes
      1. Select the “best” single-row shapes
         - Expand by adding a row above, if possible
         - Otherwise expand by adding a row below
      2. Select the “best” 2-row shapes and expand
      3. Repeat with 3-row … k-row shapes

3. Select “best” shape, remove it from the Intensity Map and repeat process until residual Intensity Map is 0.
R-squared values for linear fit

<table>
<thead>
<tr>
<th>Levels</th>
<th>numShapes</th>
<th>beamTime</th>
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</thead>
<tbody>
<tr>
<td>10</td>
<td>0.479</td>
<td>0.234</td>
</tr>
<tr>
<td>20</td>
<td>0.677</td>
<td>0.442</td>
</tr>
<tr>
<td>100</td>
<td>0.555</td>
<td>0.329</td>
</tr>
</tbody>
</table>

Basic Measures of Complexity

<table>
<thead>
<tr>
<th>Statistic</th>
<th>10</th>
<th>20</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>numNonzeroIMap</td>
<td>0.479</td>
<td>0.612</td>
<td>0.708</td>
</tr>
<tr>
<td>numNonzeroDiff</td>
<td>0.677</td>
<td>0.723</td>
<td>0.772</td>
</tr>
<tr>
<td>numNonzeroDiffVert</td>
<td>0.555</td>
<td>0.666</td>
<td>0.743</td>
</tr>
</tbody>
</table>

Complexity Affecting #Segments: counts

<table>
<thead>
<tr>
<th>Statistic</th>
<th>10</th>
<th>20</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>maxMaxUpDownSteps</td>
<td>0.489</td>
<td>0.687</td>
<td>0.702</td>
</tr>
<tr>
<td>sumMaxUpDownSteps</td>
<td>0.622</td>
<td>0.711</td>
<td>0.744</td>
</tr>
<tr>
<td>maxValleys</td>
<td>0.243</td>
<td>0.288</td>
<td>0.33</td>
</tr>
<tr>
<td>totalValleys</td>
<td>0.537</td>
<td>0.545</td>
<td>0.588</td>
</tr>
<tr>
<td>maxUpDownStepsVert</td>
<td>0.102</td>
<td>0.076</td>
<td>0.028</td>
</tr>
<tr>
<td>sumMaxUpDownStepsVert</td>
<td>0.588</td>
<td>0.665</td>
<td>0.725</td>
</tr>
</tbody>
</table>

Complexity Affecting #Segments: sums

<table>
<thead>
<tr>
<th>Statistic</th>
<th>10</th>
<th>20</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>maxSumStepsUpVert</td>
<td>0.139</td>
<td>0.128</td>
<td>0.174</td>
</tr>
<tr>
<td>sumStepsUpVert</td>
<td>0.502</td>
<td>0.649</td>
<td>0.719</td>
</tr>
</tbody>
</table>

Complexity Affecting Beam-on Time: sums

<table>
<thead>
<tr>
<th>Statistic</th>
<th>10</th>
<th>20</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>maxSumStepsUp</td>
<td>0.193</td>
<td>0.249</td>
<td>0.141</td>
</tr>
<tr>
<td>sumStepsUp</td>
<td>0.733</td>
<td>0.841</td>
<td>0.829</td>
</tr>
</tbody>
</table>

Scatter Plot Shows Linear Relationship

SumStepsUp vs. BeamTime

\[ y = 3.4678x + 307.09 \]

R\(^2\) = 0.8025

Shape Table Metaheuristic

<table>
<thead>
<tr>
<th>Shape Table</th>
<th>Metaheuristic</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
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<th>S6</th>
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<th>S21</th>
<th>S22</th>
<th>S23</th>
<th>S24</th>
<th>S25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nonzero in IMap</td>
<td>“Makes Sense”, But Has Poor Correlation</td>
<td>[Diagram of shapes and metaheuristics]</td>
<td>[Table of shape table metaheuristics]</td>
<td>[Diagram showing scatter plot with linear regression]</td>
<td>[Table summarizing R-squared values for linear fit]</td>
<td>[Table detailing complexity affecting beam-on time]</td>
<td>[Table detailing basic measures of complexity]</td>
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</tr>
</tbody>
</table>
Results: Fewer Segments!

\[ y = 14.563x + 362.4 \]
\[ R^2 = 0.829 \]

Results: Trades-off Beam-on Time

\[ y = 3.4678x + 307.09 \]
\[ R^2 = 0.8025 \]

Side-by-Side Comparison

Results: Clinical Cases Follow Similar Pattern

\[ y = 1.1965x + 19.344 \]
\[ R^2 = 0.7436 \]
Summary of Results

- 100-level discretization: 8% improvement!
- 20-level discretization: 4% improvement
- 10-level discretization: 4% improvement

Future Work

Referenced Papers


Random Intensity Map Generator: GenIMap

- Purpose:
  - Larger sample size of variable intensity shapes
  - Reduce over-tuning on our small sample set (7 IMaps!)
- Goal:
  - Produce a "realistic" representation of an intensity map considering
    - 8x24 bixel IMap
    - Tumor area 3 or more tumors (I used 5)
    - Organs at risk (3+) overlapping the tumor area
Generating the Tumor Area

- Five dense circles are created randomly within the center of IMap (8 bixels x 24 bixels with a 1-2 bixel margin on all sides).
- Each circle contains a variable radius with decreasing intensity values along the radius.
- To ensure equal spacing for these circles, they are alternately placed in the left half and right half of the IMap.

Representing the OAR

- Three smaller circles are generated and placed randomly throughout the IMap.
- These smaller circles contain a smaller variable radius with decreasing OAR density along the radius.
- To create larger, more complex problems, increase the area for the "tumor circles" and increase the number of "OAR circles."
- Keeping the element of randomness for the position and radius of each circle helps to ensure each created intensity map is distinct from others.

Ensuring Appropriateness

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  - Comparing average values of complexity measures to those of clinical cases
  - Comparing to a control set of IMaps generated using a uniform random distribution.
- Percent difference of complexity as compared to clinical cases:
  - GenIMap algorithm: 24% difference
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Radiation has been used to treat cancer for more than 100 years.

- 1896 - Emil Herman Grubbé first treats a breast cancer patient using radiation - less than one month after the public announcement of Roentgen’s discovery of X-rays.
- 1909 - Gösta Fossel first reports on the use of brachytherapy to treat 32 patients.
- 1975 - Mark P. Carol, M.D. develops initial idea for IMRT.
- 1992 - Computer technology has advanced sufficiently that Carol begins working on a prototype of the Peacock IMRT system.
- 1996 - FDA approval for Carol’s system.
Radiotherapy Motivation

• More than 1.3 million new cases of cancer each year in U.S., and many times that number in other countries

• Approximately 60% of U.S. patients with cancer have radiation therapy sometime during the course of their disease

• Organ and function preservation are important aims (minimize radiation to nearby organs at risk (OAR)).

bDiff heuristic

1. Choose weight, $\alpha$.
   \[ \alpha = \min |D_{ij}| \]

2. Create candidate shapes.
   a) create row-shapes where $D_{il+1} = \alpha$ and $D_{ir} = -\alpha$
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