

## CS 240: Introduction to Discrete Math

### Homework 1

Due Thursday September 27<sup>th</sup>.

Problem 1. How many flushes can be made from the cards in a 52-card deck (a flush is 5 cards of the same suit; there are 13 cards of each of four suits in the deck)?

Problem 2. There are 17 street lamps along a straight street. In order to save electricity and not affect the regular use at the same time, we can shut down 5 of these lamps. But we cannot turn off a lamp at either end of the street, and we cannot turn off a lamp adjacent to a lamp that is already off. Under such conditions, in how many ways can we turn off 5 lamps?

Problem 3. How many solutions are there to the equation  $x_1 + x_2 + \dots + x_n = k$  with each  $x_i$  a nonnegative integer? A positive integer?

Problem 4. How many integers from 1 through 9999 have distinct digits? How many contain exactly one 7 and one 9? Note that digit strings with leading zeroes (like 085) are not considered valid integers.

Problem 5. Consider a 20-man race in which the top three finishers receive gold, silver, and bronze medals respectively, and the rest of the top 10 finishers get a consolation medal. How many distinct finishes are there to the race (in terms of who receives which medal)?

Problem 6. Give a proof that  $k \cdot C(n, k)$  equals  $n \cdot C(n-1, k-1)$ .

Then calculate the SUM  $k = 1$  to  $n$  of  $k \cdot C(n, k)$ .

Hint: The binomial theorem with  $x=y=1$  says that SUM  $i = 0$  to  $n$  of  $C(n, i) = 2^n$