

Homework 3

1 Lewis Carroll's Syllogisms

Below is one of Lewis Carroll's Syllogisms. Translate each of his statements (lines) into statements in Propositional Logic. Define carefully what each of your variables means and the universe of discourse. Then write the contrapositive for each statement. Finally, using transitivity of implication what can you conclude?

No birds, except ostriches, are 9 feet high.

There are no birds in this aviary that belong to anyone but me.

No ostrich lives on mince pies.

I have no birds less than 9 feet high.

2 Conjunctive and Disjunctive Normal Forms

In this problem we look at Propositional Statements that have very special forms. To begin with we need some definitions.

A *literal* is just a variable or the negation of a variable. So, something like A or $\neg A$.

A *conjunction* is a Propositional statement that is an "and" of literals. So for example, $A \wedge \neg B \wedge C \wedge \neg D$.

A *disjunction* is a Propositional statement that is an "or" of literals. So for example, $A \vee \neg B \vee C \vee \neg D$.

A Propositional statement is in *conjunctive normal form* if it is a conjunction of disjunctions. For example, $(A \vee \neg B) \wedge (B \vee C \vee \neg D) \wedge (A \vee C)$.

A Propositional statement is in *disjunctive normal form* if it is a disjunction of conjunctions. For example, $(A \wedge \neg B) \vee (B \wedge C \wedge \neg D) \vee (A \wedge C)$.

The Propositional statements A , $\neg A$, $A \vee B$ and $A \wedge B$ are in both disjunctive and conjunctive normal form. Be sure you understand why this is true.

Notice that if a Propositional statement is in conjunctive or disjunctive normal form then any negations appear directly in front of a variable. For example, $\neg(A \wedge \neg B) \wedge (C \vee D)$ is a Propositional statement, but its not in conjunctive normal form or disjunctive normal form. But it is equivalent to $(\neg A \vee B) \wedge (C \vee D)$, which is in conjunctive normal form.

Fact: it is possible to rewrite any Propositional statement into an equivalent statement that is in conjunctive (or disjunctive) normal form. This is done by repeated application of the distributive law, DeMorgan's laws, and the law of double negation ($\neg\neg A$ is equivalent to A).

Part A For each of the following statements find an equivalent statement in conjunctive normal form.

- 1) $\neg(A \vee B)$
- 2) $\neg(A \wedge B)$
- 3) $A \vee (B \wedge C)$

Now let's turn our attention to disjunctive normal form.

Part B For each of the following statements find an equivalent statement in disjunctive normal form.

- 1) $\neg(A \vee B)$
- 2) $\neg(A \wedge B)$
- 3) $A \wedge (B \vee C)$

Part D Describe a method for converting any Propositional statement into an equivalent Propositional statement that is in conjunctive normal form.

Part E Describe a method for converting any Propositional statement into an equivalent Propositional statement that is in disjunctive normal form.

3 Propositional Statements that use only NAND

The propositional connective NAND (not AND) is defined by the following truth-table.

Table 1: NAND

A	B	A NAND B
T	T	F
T	F	T
F	T	T
F	F	T

The goal of this problem is to show that every statement in Propositional Logic is equivalent to a statement that uses only the logical connective NAND.

To approach this problem show that the statements $\neg A$, $A \wedge B$, and $A \vee B$ are equivalent to statements that just use NAND.

Now explain how more complicated statements can also be translated into equivalent statements just using the connective NAND. Illustrate your method using the following statement:

$$(((A \wedge B) \vee (B \wedge \neg C)) \vee \neg D) \wedge E$$