Predicates and Predicate Logic

Motivating example: Consider

If \((x + y \geq 300)\) then \((x \geq 150 \text{ or } y \geq 150)\)

Is this true or false?

Suppose we let \(P : x + y \geq 300\)
\(Q : x \geq 150\)
\(R : y \geq 150\)

Then \(P \implies Q \lor R \equiv\)

Predicates

decalate : a mapping from some underlying domain \(D\) to propositions

Examples:

\(P(x) : x^2 \geq x\) \hspace{1cm} \text{domain is}\n\(Q(x) : x = x + 1\) \hspace{1cm} \text{domain is}\n\(S(x) : x \text{ ends in the letter 'y'}\) \hspace{1cm} \text{domain is}\n\(T(x) : x \text{ has at least 9 letters}\) \hspace{1cm} \text{domain is}\n\(\text{Even}(x) : x \text{ is even}\) \hspace{1cm} \text{domain is}\n
\(\text{Costars}(a, b, m) : a \text{ and } b \text{ both appeared in movie } m\)
\(\text{domain of } a, b \text{ is}\)
\(\text{domain of } m \text{ is}\)

Creating propositions from predicates:

1.
2.

Quantification

Universal quantification

\((\forall x) P(x)\) means
\(\circ true\) if
\(\circ false\) if
If domain is unclear, specify it:
Common numeric domains:

Examples
Suppose we have the domain Days = {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}

\((\forall x \in \text{Days}) \ S(x)\)

\[\equiv S(\text{Monday}) \land S(\text{Tuesday}) \land S(\text{Wednesday}) \land S(\text{Thursday}) \land S(\text{Friday}) \land S(\text{Saturday}) \land S(\text{Sunday})\]

\((\forall x) \ P(x)\) in other words,

**Existential quantification**

\((\exists x) \ P(x)\) means

- **true** if
- **false** if

Examples

\((\exists x \in \text{Days}) \ T(x)\)

\((\exists x) \ Q(x)\)

**Relationship between universal and existential quantification**

\(\neg (\forall x) \ P(x) \equiv \neg (\exists x) \ P(x)\)

To get a feel for why, suppose \(D = \{x_0, x_1, x_2, \ldots, x_n\}\)

Then \((\forall x) \ P(x) \equiv \)

and \((\exists x) \ P(x) \equiv \)