Predicates and Predicate Logic

Motivating example: Consider

If $(x + y \geq 300)$ then $(x \geq 150$ or $y \geq 150)$

Is this true or false?

Suppose we let P : $x + y \geq 300$
Q : $x \geq 150$
R : $y \geq 150$

Then $P \Rightarrow Q \lor R \equiv$

Predicates

predicate : a mapping from some underlying domain D to propositions

Examples:

P(x) : $x^2 \geq x$
Q(x) : $x = x + 1$
domain is

S(x) : x ends in the letter 'y'
T(x) : x has at least 9 letters
domain is

Even(x) : x is even
domain is

Costars(a, b, m) : a and b both appeared in movie m

domain of a, b is

domain of m is

Creating propositions from predicates:

1.

2.

Quantification

Universal quantification

$(\forall\ x)$ P(x) means

- *true* if
- *false* if

If domain is unclear, specify it:
Common numeric domains:

Examples

Suppose we have the domain Days = {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}

$(\forall\ x \in \text{Days})\ S(x)$

$\equiv S(\text{Monday}) \wedge S(\text{Tuesday}) \wedge S(\text{Wednesday}) \wedge S(\text{Thursday}) \wedge S(\text{Friday}) \wedge S(\text{Saturday}) \wedge S(\text{Sunday})$

$(\forall\ x)\ P(x)$ in other words,

Existential quantification

$(\exists\ x)\ P(x)$ means

- *true* if
- *false* if

Examples

$(\exists\ x \in \text{Days})\ T(x)$

$(\exists\ x)\ Q(x)$

Relationship between universal and existential quantification

$\neg\ (\forall\ x)\ P(x) \equiv$

$\neg\ (\exists\ x)\ P(x) \equiv$

To get a feel for why, suppose $D = \{x_0, x_1, x_2, ..., x_n\}$

Then $(\forall\ x)\ P(x) \equiv$

and $(\exists\ x)\ P(x) \equiv$