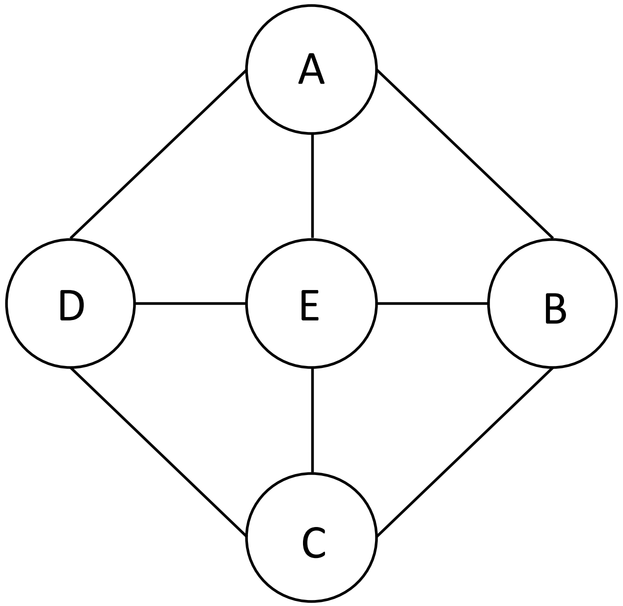
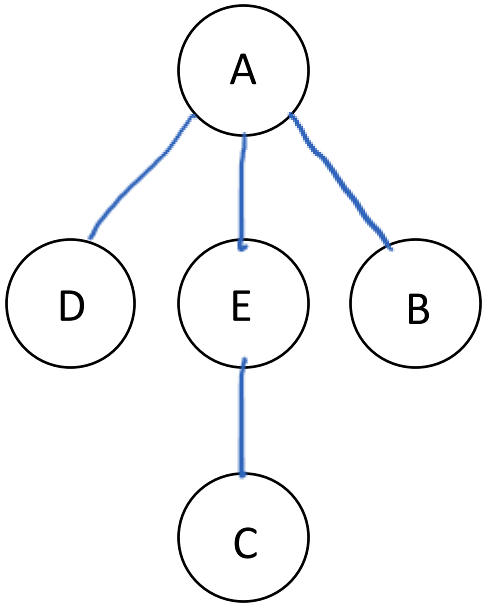
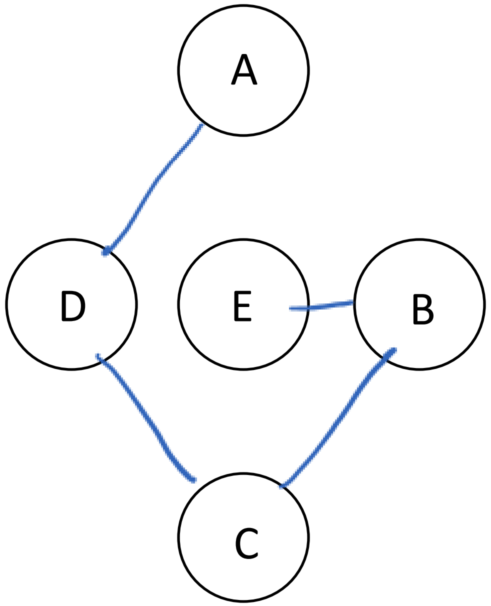
Spanning Trees

Definition

For an undirected graph G with V nodes: If T is a subgraph of G and T contains all nodes of G, is connected, and has exactly (V-1) edges, then T is a spanning tree of G.







Depth First Spanning Trees

DFT(v):

mark v as visited

for each unvisited successor u of v:

add v - u to spanning tree

DFT(u)

Breadth First Spanning Trees

```
BFT(v):
        q = new Queue()
        mark v as visited
        q.enqueue(v)
        while (!q.isEmpty()):
                c = q.dequeue()
                for each unvisited successor u of c:
                        add edge c – u to spanning tree
                        mark u as visited
                        q.enqueue(u)
```

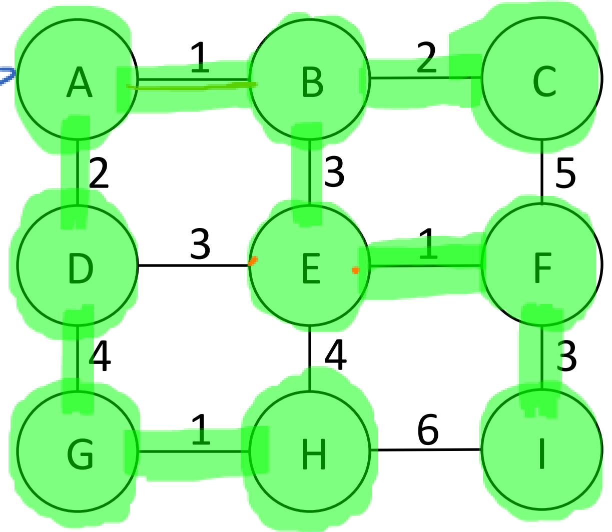
Minimum Spanning Trees

In a weighted graph:

Minimum spanning trees are the spanning trees with the lowest sum of edge weights.

Prim's Algorithm

Start node



For weighted, connected, and undirected graphs.

start node

```
prim(v):
    pq = new PriorityQueue()
    mark v as visited
    pq.insert(v.outgoingEdges)
    while (!pq.isEmpty()):
        c = pq.removeMin()
        if (c.endNode is unvisited):
            mark c.endNode as visited
            add c to tree
            pq.insert(c.endNode.outgoingEdges to unvisited nodes)
```

$E \cdot \log E$

$\rightarrow$ E iterations

$\rightarrow \log E$

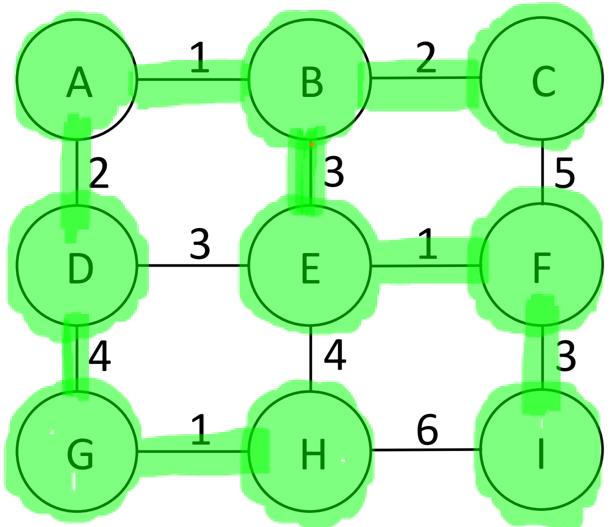
$\hookrightarrow \log E$

$\rightarrow E \cdot \log E + E \cdot 2 \cdot \log E = 3 \cdot E \cdot \log E$

p.q.: ~~AB:1~~, ~~AD:2~~, ~~BC:2~~, ~~BE:3~~, ~~DE:3~~, ~~DG:4~~, ~~CF:5~~, ~~EF:1~~, ~~EH:4~~, ~~FI:3~~, ~~IH:6~~, ~~GH:1~~

$\sum_{\text{edge weights}} = 17$

Kruskal's Algorithm



For weighted, connected, and undirected graphs.

edgeList: ~~AB:1~~, ~~EF:1~~, ~~GH:1~~, ~~BC:2~~, ~~AD:2~~, ~~BE:3~~, ~~DE:3~~, ~~FI:3~~, ~~DG:4~~, ~~EH:4~~, ~~CF:5~~, ~~HI:6~~

```
kruskal(edgeList):
    sort edgeList
    init nodeSets with singleton sets
    while (!edgeList.isEmpty()):
        c = edgeList.removeFirst()
        if (c.startNode and c.endNode in different nodeSets):
            add c to tree
            nodeSets.join(c.startNode, c.endNode)
```

sort edgeList → $E \cdot \log E$

init nodeSets with singleton sets → V

while (!edgeList.isEmpty()) → E iterations

if (c.startNode and c.endNode in different nodeSets) → $\log V$

nodeSets.join(c.startNode, c.endNode) → $\log V$

$$\sum_{\text{edgeWeights}} = 17$$

$$\Rightarrow E \cdot \log E + V + E \cdot 2 \cdot \log V$$

Complexities

V: # nodes in graph

E: # edges in graph

Prim: $O(\cancel{3} \cdot E \cdot \underline{\log E})$

$O(E \cdot \log V^2)$

$O(E \cdot \cancel{2} \cdot \log V)$

Kruskal: $O(E \cdot \underline{\log E} + \cancel{V} + 2 \cdot E \cdot \log V)$

$O(E \cdot \log V^2 + 2 \cdot E \cdot \log V)$

$O(\cancel{4} \cdot E \cdot \log V)$