Untangling Quadrilateral Meshes using Locally Injective Mappings

Chaman Singh Verma, Krishnan Suresh
University of Wisconsin, Madison, WI 53706

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Problem

Mesh tangling is a common occurrence in mesh generation, deformation, and morphing.

1. Tangled meshes are not suitable for numerical simulations.
2. Tangled meshes create artifacts in texture mappings.

Mesh untangling is essential.

Current methods

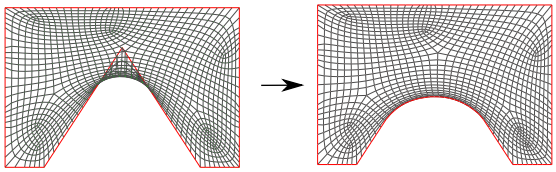
Almost all existing methods pose untangling as a non-linear optimization problem and move internal nodes. In general, these methods are (1) **expensive,** and (2) have ***no convergence guarantees.***

Observation: Tangling usually occurs near the concave regions.

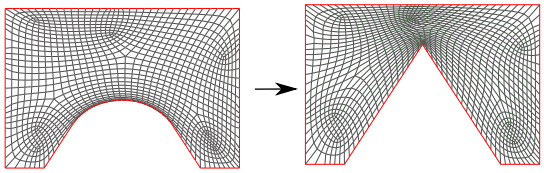
Many linear and non-linear Laplacian Smoothing or Lloyd Relaxation operators produce tangle-free meshes for weakly convex domains.

Our Method

Step 1: Convexification: Smooth concave corners using $V_i = 0.5(\ _{i-1} + V_{i+1})$ until all the mesh elements become positive.



Step 2: Locally Injective Mapping: The modified boundary is morphed into the original shape with non-inversion constraints.



Minimize Energy of distortion caused by:

1. Distance between source and target shapes.
2. Distance of internal nodes from ideal locations.

$$E = E_{pos}(f) + \lambda E_{reg}(f)$$

Constraint: All elements must be non-negative.

Advantages

1. It works for large class of problems.
2. Simple to implement.
3. Extendable to hexahedral mesh.

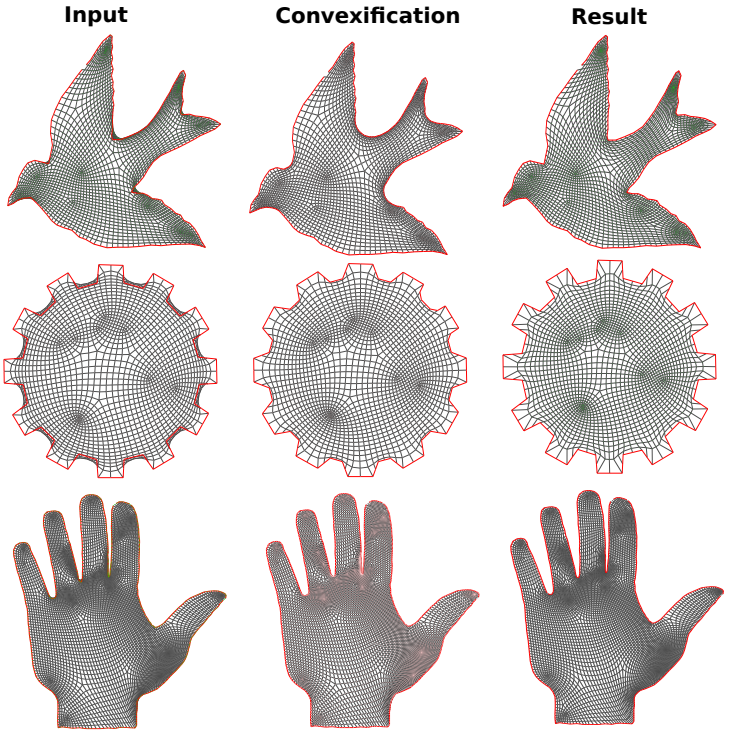
Untangling test

Knupp's Statement: There is no known a priori test to determine, if a given mesh can be untangled.

Proposition-I: If a mesh remain untangled in the convex domain,then the mesh topology must be incorrect and it can never be untangled in the non-convex domain.

Proposition-II: If a mesh has unrecoverable sliver elements before shape recovery using LIM, then it is unlikely to have untangled mesh in original geometry.

Results

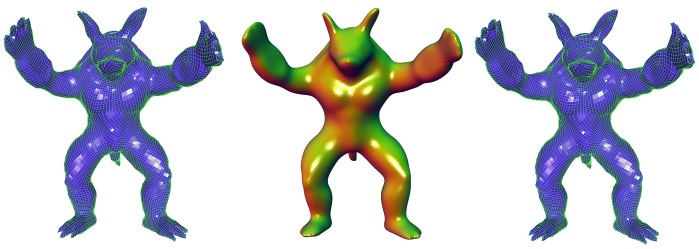


Performance

Model	Inverted/Total Elements	Scaled Jacobian	Boundary modification (sec)	Morphing (sec)
Bird	164/2402	-0.99/0.40	4.5	2.8
Wheel	198/1232	-0.94/0.32	4.9	2.5
Hand	74/6484	-0.96/0.30	24	8.3

Extension to 3D

Generalization to Hex mesh



Mean Curvature Flow/Spin Transformation(Keenan Crane, 2013)

(Quadmesh source: Dr. David Bommes)

References

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