CS 547: Computer System Modeling Fundamentals

Fall 2006

Practice Problems for the Final Exam

1. Consider an M/G/1 queue with arrival rate equal to 10 customers per minute and deterministic service times equal to 5 seconds.

   (a) What is the server utilization?

   (b) What is the average queue length?

   (c) What is the average number of other customers in the queue when a customer enters service?

2. Consider two M/G/1 queues, each with arrival rate λ and mean service time equal to S. If the first queue has deterministic service times, and the second queue has exponential service times, what is the ratio of the expected waiting time in each queue, where waiting time does not include the time in service?

3. Consider an M/M/1 queue with arrival rate λ and mean service time $1/\mu$. Let s denote the actual service time of a customer A that arrives to the queue. (Note that s is a constant, not a random variable.) Derive:

   (a) Customer A's expected waiting time (before receiving service).

   (b) Customer A's expected residence time (i.e., waiting time plus service time).

   (c) The average number of customers that arrive during an arbitrary time interval of length t.

   (d) The expected number of other customers that arrive while the customer A is waiting to receive service.

   (e) The expected number of *other* customers in the queue at the instant when customer A enters service.

   (f) The expected number of customers that arrive while customer A is in service.

   (g) The expected number of customers that are in the queue when customer A leaves service.

4. Give an equation expressing the mean residence time R as a function of the mean total service time S and the utilization U ($=\lambda S$), for each of the following systems.

   (a) A FCFS single server system with Poisson arrivals and independent, exponentially distributed service times.

   (b) Like part (a), but instead of exponentially distributed service times, customers have service times uniformly distributed between 0 and 2S.

   (c) Like part (b) (i.e., uniformly distributed service times), but customers arrive in pairs (i.e., the bulk arrival rate is $\lambda/2$, and each bulk has exactly two customers).

(d) Like part (a), but customers have deterministic service times, and there is feedback: each customer, after completing service for the first time, rejoins the end of the queue. After completing service for the second time, the customer departs the system. For each of these two visits the service time is exactly S/2.

5. Consider a server that processes two types of customers. Type A customers have a deterministic service time equal to 1 second; type B customers have a deterministic service time equal to 6 seconds. Customers arrive according to a Poisson process of rate 0.5 customers/second, with no correlations between the types of successive customers or between customer types and interarrival times. 90% of customers are of type A, and 10% are of type B. Give (i) the mean residence time for type A customers, (ii) the mean residence time for type B customers, and (iii) the overall mean customer residence time, under each of the following scheduling disciplines.

   (a) FCFS (every customer joins the end of the queue upon arrival, and is served in FCFS order)

   (b) LCFSPR ("Last-Come-First-Served-Preemptive-Resume"): at each point in time, the customer with the *latest* arrival time, among those customers present at the server, has preemptive priority over all other customers.

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