Problem 1: 15 points (15=3*5) Remember to give a reason(ing) for your answer.

T F ? The third divided difference $f[x_1, x_2, x_3, x_4]$ for $f(x) = \sin(x) - 1$ is positive for every choice of points x_1, x_2, x_3, x_4.

T F ? $(.1)_4 = (.020202...)_3$ (a repeating fraction).

T F ? $\int_a^b f(x)\,\mathrm{d}x = (b-a)(f(a) + f((a+b)/2) + f(b))/3 + (1/12)(b-a)^2 f''(\xi)$ for all polynomials f and with ξ some point between a and b.

T F ? `randn(m,n)` returns an m-by-n matrix whose entries are randomly chosen from the interval $[0\,..\,1]$.

T F ? There are many cubic polynomials that agree with the function $f(x) = 1 + \sin(x)$ at the three points $-\pi$, 0, and π.

Problem 2: 20 points (a) Use a divided difference table to construct the cubic polynomial, p, that satisfies the following four conditions: $p(-1) = -1$, $p'(-1) = 2$, $p(2) = -4$, $p''(-1) = -2$, in Newton form.

(b) Use Nested Multiplication to evaluate the polynomial p at 0.

(c) Show that, without any further computation, one can read off from the p constructed the *quadratic* polynomial, q, that matches the information $q(-1) = -1$, $q'(-1) = 2$, $q(2) = -4$.

Problem 3: 15 points Rewrite the following script to make it as efficient and as loop-free as possible. (Assume that `n`, `k`, and the array `c` of size `[n,k]` are already defined.)

```
for i=1:n
   for j=1:k
      c(i,j) = (k-j)*c(i,j);
   end
end
```

Problem 4: 15 points Construct the natural cubic spline interpolant to $f(x) = \sin(x)$ at the point sequence `linspace(0,pi,2)`. Be sure to explain your answer carefully.

Problem 5: 15 points Using 2-(decimal)-digit floating point arithmetic *throughout*, compute $1 - \sqrt{.99}$ correctly to 2 significant (decimal) digits. (Since it is hard to compute squareroots by hand, you may use the fact that, to 2 significant decimal digits, $\sqrt{.99} = .99$. But be aware that, because of loss of significance, the straightforward answer $.01 = 1 - .99$ is not even right in the first significant digit.)

(one more problem, on the next page)

Problem 6: 20 points The composite Midpoint rule for approximating the integral

$$\int_a^b f(x)dx$$

has error equal to $(1/24)(b-a)h^2 f''(\xi)$ for some ξ between a and b. Suppose you are required to compute the integral

$$\int_0^4 \ln(1+x)\,\mathrm{d}x$$

with an error no bigger than 10^{-4} using the composite Midpoint rule. What is the biggest h you could use?